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SEPTEMBER, 1932

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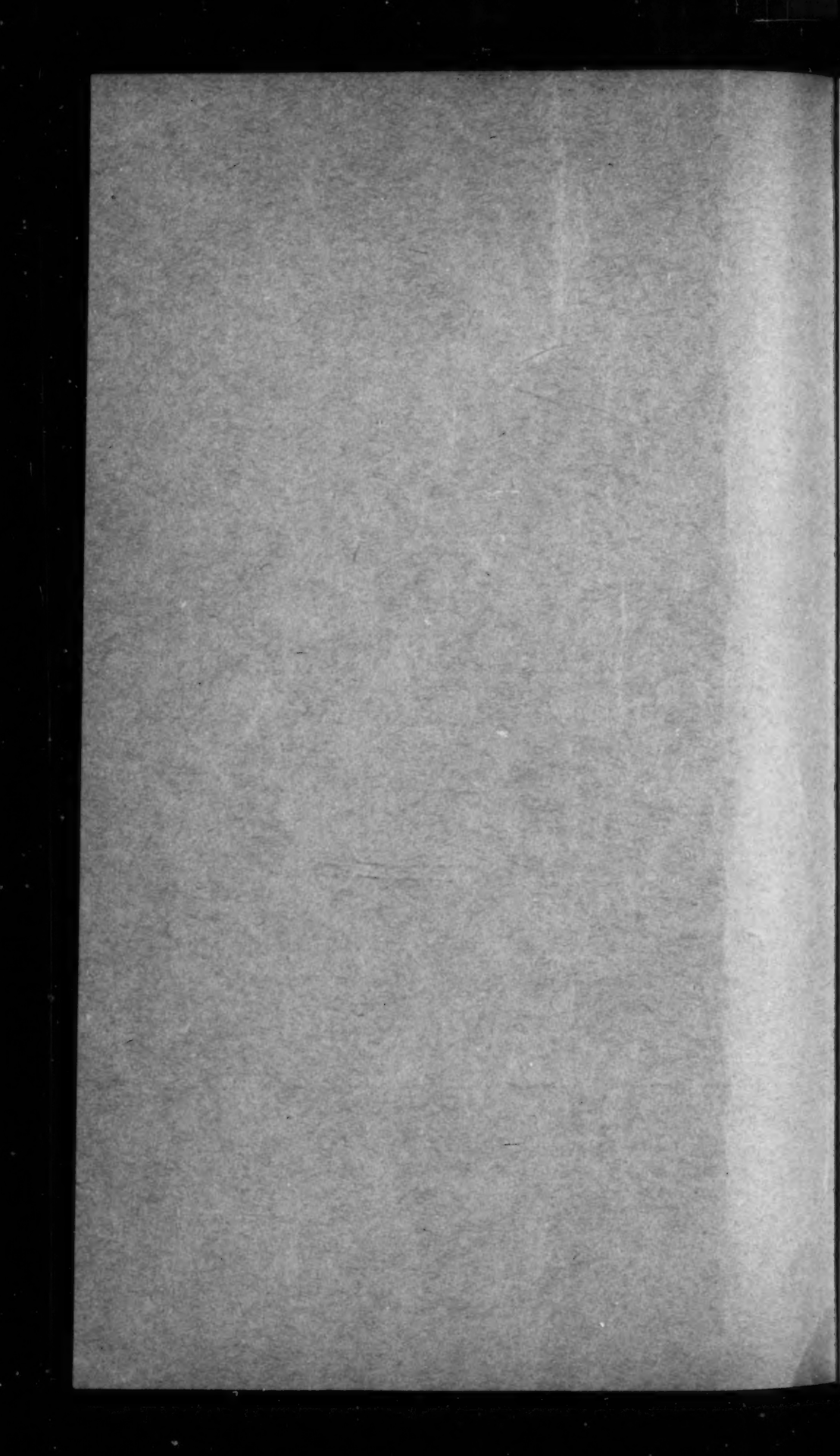

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JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

GOLD, INTERNATIONAL CREDITS AND DEPRESSION[1]

By James Harvey Rogers

PART I

British economists have frequently pointed out important relationships between the international gold situation and the current depression. In amplified form their argument is usually somewhat as follows.

In gold standard countries, the monetary units are defined as given weights of pure gold. In the United States, for example, the dollar is defined as 23.22 grains of fine gold. Moreover, having been thus defined, careful provision has been made to maintain the definition. If anyone has gold, he may, by taking it to one of the government offices, receive in return dollars, one for each 23.22 grains of fine gold. If, on the other hand, he has dollars, he may demand gold coins or gold bars in exchange and thus receive 23.22 grains of fine gold for each dollar presented.

In other words, gold and dollars are kept completely interchangeable —23.22 grains of fine gold being kept always interchangeable with one dollar.

Because of this simple but invariable relationship, it follows that the value of this given weight of gold is always exactly equivalent to that of a dollar and vice versa. Whatever happens, therefore, to the value of gold happens automatically to that of the dollar and whatever happens to the value of the dollar happens equally automatically to the value of gold.

Suppose, for example, that gold should become as plentiful and as cheap as copper, the dollar—important lags for the moment being disregarded—would become equally cheap and would buy correspondingly little. If, on the other hand, gold were to become greatly in demand

[1] Part I of this paper was delivered before the Annual Meetings of the American Statistical Association in Washington, December 30, 1931. Part II has been added as of June 15, 1932.

without equally great increase in the supply, its value relative to that of other things would rise correspondingly, and so would that of the dollar.

That there is just such a greatly increased demand for gold in the world today and that the supplies available for those needing it most urgently have at the same time become more difficult to obtain, I shall now proceed to add to the usual argument just given. In fact, among the civilized nations of the world there is a veritable scramble for gold and the drastic price declines from which, for more than two years, the world has been suffering, is in no uncertain way connected with this scramble.

Of the approximately 400 million dollars worth of gold mined each year, roughly one-half goes into the arts to be made into watches, table ware, and jewelry, and to be used in filling teeth. The other half goes automatically into the monetary gold stock of the world—there to serve as a basis for our money and credit systems. This enormous gold reserve now amounts to a little more than eleven billion dollars.

Where, then, you may ask, is there any possible shortage? The answer is: There is no actual shortage for all the gold-standard world taken together, but this huge existing stock is so badly distributed that while, in two countries, large unused supplies lie locked up and idle, in many others, there is too little to maintain the standard.

The United States, for example, even after its recent great losses, holds 36 per cent of the world's total and France 22 per cent. England, on the other hand, holds barely 6 per cent and Germany less than 3 per cent. Moreover, a situation very similar to the one just described has existed for several years, with the United States and France claiming an ever larger portion of the total. The charts give in more detail the present (March 31, 1932) character and continued growth of this extraordinary maldistribution.

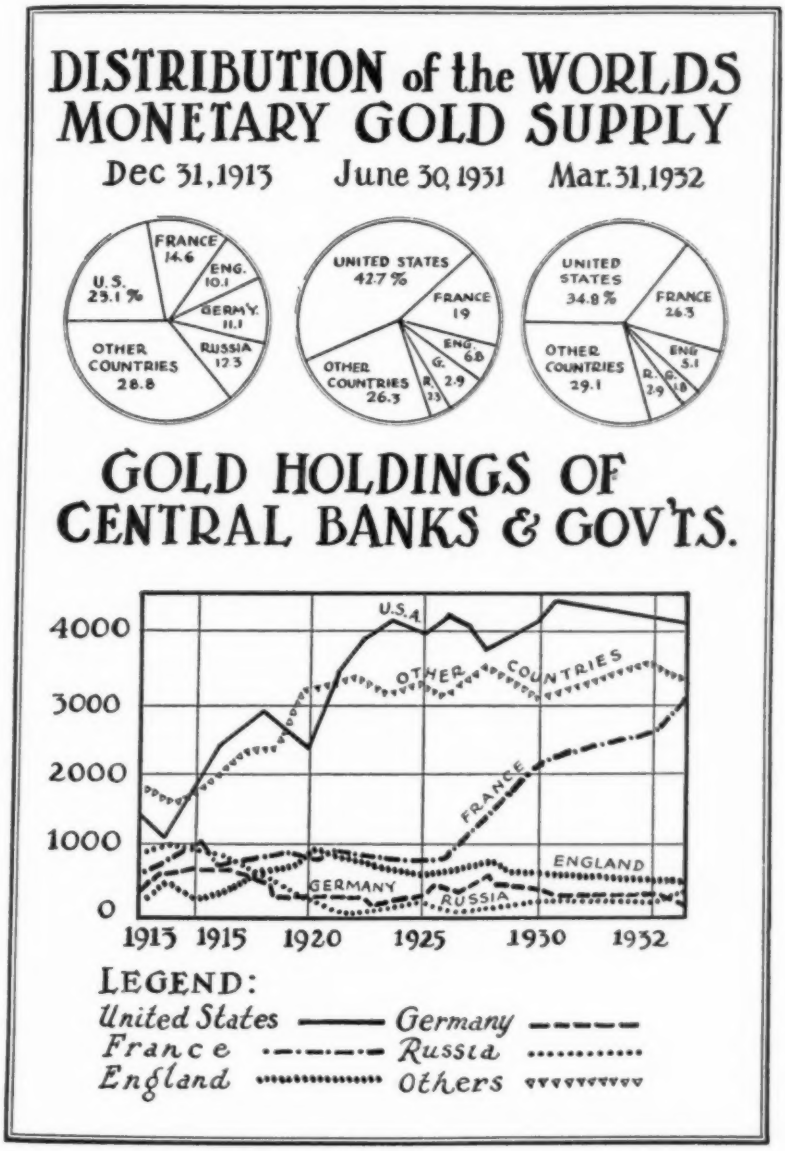
The difficulties experienced in emergency by a country suffering from a gold shortage are perhaps best exemplified by those recently confronted in England and in Germany.

In England, during a number of months prior to the collapse of the gold standard, it became more and more generally known that the entire British gold stock was less than enough to pay the short-term credits held in London by Paris bankers alone.

In Germany, at the same time, it became known that the total of foreign short-term credits in the Berlin market was more than five times the gold holdings of the central bank of that country.

Since a great portion of such short-term credits can be withdrawn literally overnight, the danger of being thrown suddenly off the gold

CHART I



standard was continually imminent in both of the countries mentioned. Moreover, in all the big money and investment centers are sold many internationally-held securities. Once a gold standard is threatened, the sale of securities in the distressed country and the purchase of those

payable in more certainly stable money units increases the ever more dangerous gold outflow. Under such circumstances, the possibilities of political pressure from abroad are as evident as are the uncertainties in the credit structure and in the securities' markets of the threatened country.

The evils of a gold shortage, therefore, while often unnoticed until some jolt to international confidence appears, are thus ever-present in the form of a threat to economic, to political, and even to diplomatic security. Strenuous and persistent efforts therefore are made to remove them.

The quickest and often the easiest way to build up a waning gold reserve is, through central bank action, to stimulate a rise in short-term interest rates. Funds are thus attracted from abroad and usually bring with them a gold inflow. In periods of stress, however, when confidence has already been shaken, not only does such a device often fail to bring the expected relief, but whatever relief is thus found intensifies the threat of short-term credit withdrawal.

More permanent and immediately less dangerous devices are therefore sought. If exports can be increased relatively to imports much more stable and satisfactory relief is found. Hence strenuous efforts are made to throw all the goods possible on the international markets and to take as few as possible therefrom.

To stimulate exports, bounties and dumping of one sort or another are instituted.

For reducing imports, the most ready device is the protective tariff. Moreover, in periods of falling prices, like the present one, it is especially popular because it gives *apparent*—though usually only apparent—promise of raising directly internal prices at the same time that it attracts gold from abroad. Hence, in the period of our history when international communications have made their greatest advances, the world is fast becoming a series of walled estates not unlike those of the Dark Ages.

To gain additional control over international trade balances and hence over gold movements, many countries have gone further still. Central banks have been given more and more complete control over the foreign exchanges and many imports are arbitrarily cut off by a refusal to grant the exchange required for payment. To these have been added no less damaging "*import quotas.*"

It is to such vicious and highly damaging mediaeval policies that the gold scramble, in combination with other influences, is leading. Meanwhile, the price level has continued its tragic decline and the business of the world has become more and more depressed.

In the United States, the gold scramble has taken several unusual and, I am afraid, unnoticed forms.

The migration of American industry, which by the end of 1930 had already reached appalling proportions, seems to be increasing at an accelerated rate. Certainly, the growth of ever higher tariffs and other trade restrictions in many parts of the world—notably in the British Empire—have provided ample encouragement. Thus, during the deepest depression of which we have reliable records—while American raw materials, of their usual high quality, flood our markets and American laborers, as dependable, as efficient and as anxious to work as ever, clamor for non-existent jobs—American firms with gigantic purchasing and employing power are fast seeking new homes in foreign lands.

Meanwhile, with the continued scarcity of so-called "free gold" in our banking structure, Federal Reserve credit policies—which, throughout the depression have been at least halting and spasmodic in the direction of monetary ease—seem to have taken an abrupt change toward relative tightness. Thus, with 36 per cent of the world's total gold stock we seem to have entered directly the scramble for a still larger share of this much sought metal.

Under such circumstances not only are we no longer exerting our influence to relieve the world's distressing monetary conditions, but our ponderous weight bids fair to be added to the already grave pressure toward further price declines.

The relation between falling prices and depression requires but little comment. As business is at present organized, costs in general precede selling prices. If, during the process of production, selling prices so decline as to fail to meet previously incurred costs, profits are automatically turned into losses and many businesses are forced to close.

One other often forgotten influence is of equal importance. Far from being entirely flexible and hence adjustable to changing conditions of all sorts, our price system contains prices of every degree of rigidity. On one end of the scale are those completely flexible, adjusting themselves readily to happenings, or even rumors of happenings, in any part of the world. On the other end, are those of almost complete inflexibility, changing only to meet the rulings of a bankruptcy court. If the price of your product is among the highly flexible ones while your costs —as is frequently the case—are highly inflexible, your profits, in periods of rapidly falling prices, are likewise automatically turned into losses. And the more rapid and more drastic the declines, the greater and more general will the losses be.

Whatever may be said, therefore, as to the so-called causes of the

depression, of this we may be confident: *Until the drastic and rapid decline of commodity prices can be at least arrested, business recovery on a broad scale can hardly be anticipated.* In consequence, it is of the utmost importance that potent measures to this end be adopted.

If the gold scramble is to be relieved, the existing supplies in France and in the United States, where plentiful, will have to be more effectively used.

To be more effectively used, measures must be taken to make them support a greater credit and money super-structure.

Very fortunately, our excellent Federal Reserve System provides the machinery for just such measures. Through open-market purchases of bills and of government securities, if maintained courageously and with persistence, these central banking institutions have it within their power to bring early, if only partial, relief.

Such purchases put entirely new funds into the hands of the sellers. If the seller is a banker, he thus increases directly his lending and investing power; if an individual (or firm) he deposits the proceeds in his bank which thus gets a similar though indirect enhancement of its lending position. Even if—as frequently happens—the receiving bank uses the new funds only to pay existing indebtedness at the Federal Reserve Bank or with a city correspondent, the enhancement of its lending position—and more important still—of its power to command the confidence of its depositors is just as great. Moreover, in a period of disastrous liquidation, like the present one, the receipt of such additional funds in the form of new deposits removes for many hard pressed banks the painful necessity of selling their best and most liquid investments on a weak and declining market.

Unfortunately, however, one dangerous obstruction to the pursuance of such a policy has made its appearance. Directly connected with the international gold scramble above described, it seems to have appeared in more nebulous and hence more insidious form to those charged with the operation of our money and credit systems. Indeed, one cannot but wonder whether the sometimes ingenious but usually unconvincing rationalizations of a "do-nothing" policy on the part of our central-banking authorities in the present emergency do not—often unconsciously perhaps—merely cloak this important but little understood restrictive influence.

As mentioned above, the "free gold" in our Federal Reserve Banks, during the past few months has averaged but little more than half a billion dollars. Thus, at a time when our monetary gold holdings amount to more than 35 per cent of the entire world stock, our credit policies are actually being gravely influenced by a virtual gold shortage.

How is it possible for such an extraordinary state of affairs to arise?

In our Federal Reserve System, gold has *two* important uses. The first is as reserve—40 per cent gold against notes and 35 per cent gold or other "lawful money" against deposits.

The second is just as simple though perhaps less generally understood. Federal Reserve Notes must be collateraled either by "elligible paper" ("Bills discounted" and "Bills bought on the open market" [1]) or else by gold. The fact that the gold thus pledged against notes can be counted likewise as reserve does not under present extraordinary circumstances relieve the severe strain which this often forgotten use places upon our gold supplies. Indeed, it so happens that in recent months "elligible paper" in the Federal Reserve Banks has been so scarce that a very large proportion of our Federal Reserve Notes outstanding have been necessarily covered by gold. Moreover, any additional issue of such notes, unaccompanied by a like increase in "elligible paper" in the vaults of the Federal Reserve Banks would further reduce—dollar for dollar—our small and fast waning "free gold" stock.

In such a situation, the large foreign balances held in New York have given natural but often undue concern. The possibility, through the recall of such credits, of a sudden withdrawal of upwards of 600 million dollars by the French—and perhaps of an equal amount by all others together—has at times been disquieting. Nor does the fact that a considerable portion of these funds are at present invested in bankers' acceptances bought on the New York market completely allay well-founded fears. While such "bills" released by foreign holders would undoubtedly be purchased in large part by the Federal Reserve Banks, which would in this way restore the available "free gold" to the extent of such purchases, the net loss might prove disturbing.

Two other features of the situation bring no reassurance. The large holdings abroad of American and of other securities salable on the New York market furnish an added threat in case confidence in our ability to maintain the gold standard should further wane. Heavy sales of such holdings would bring an added draft upon our gold supply.

Would such a draft be accompanied by a release of roughly equivalent amounts of "free gold"? The answer is: Under present circumstances it probably would be—especially if the withdrawals were sudden.

The reason is that such withdrawals would be largely counterbalanced by accompanying forced "rediscounts" on the part of member banks faced with the necessity of providing the funds being turned into gold for export. Should the security sales be gradual and slow, how-

[1] These terms are used in the special senses given them in the statements of the Federal Reserve Banks.

ever, hard-pressed banks would probably continue their recent policy of meeting many such demands by a further liquidation of their loans and investments. Thus, from a double source, would the situation in the securities markets be further aggravated, and general credit continue its rapid contraction.

Should panic conditions result, even a "flight from the dollar" by Americans is conceivable, and hoarding would certainly increase greatly. Moreover, while each of these drains on our gold supply would doubtless be to a considerable extent counteracted by a further expansion of forced "rediscounts" and hence by corresponding increases in available "free gold," the alternative further contraction of loans and investments on the part of many banks meeting the withdrawals would continue the vicious spiral of liquidation.

Under such circumstances, large purchases of government securities by the Federal Reserve Banks—unless effective in restoring business health with entirely unhoped for rapidity—would certainly reduce further the "free gold." The reason is not far to seek.

Almost invariably such purchases have in the past led to a similar reduction in "rediscounts" on the part of the member banks. In other words, the new funds put out to the banks through open-market purchases, have been in general returned by them through paying of their indebtedness at the Federal Reserve Banks. Hence, while there is no doubt that such cancellations of indebtedness improve the lending position—as well as disposition—of the bankers concerned, it is equally evident that the loss of "elligible paper" by our central institutions reduces by a like amount the available "free gold."

Hence, under current conditions, further purchases of government securities by the Federal Reserve Banks would increase their demand obligations at the same time that it would decrease their capacity for meeting them.

Thus with our enormous gold holdings we find ourselves in the preposterous and—were it not so tragic—ludicrous position of having to adapt our internal credit policies to the demands of a threatened gold shortage. On this account the most constructive proposal yet made for relieving domestic as well as international money and credit conditions has at least temporarily been precluded. And the United States has actively entered the gold scramble.

Meanwhile the gradual drift of world economic affairs under a passive credit policy in this country is to a no less tragic and uncertain state.

To dissipate once and for all this extraordinary "gold scare" and to make possible again the free and intelligent utilization of our enormous gold stocks, the means are as simple as they should be easy of attain-

ment. What is alone needed is that in emergency the "cover" requirements for our Federal Reserve Notes be liberalized. Just as it has been provided from the beginning that all *reserve* requirements may at the discretion of the Federal Reserve Board be suspended for thirty days and re-suspended as often as required for fifteen days at a time, so a similar provision should be incorporated making legal the emergency use of government securities as "cover" for Federal Reserve Notes. Certainly, it is preposterous to provide carefully against embarrassment from a sudden temporary shortage of gold in one of its two uses and at the same time to leave ourselves completely unprotected from the dangers of a similar shortage in the other more rigid and more exacting use.

Some such modification of the Federal Reserve Act is the *sine qua non* of any constructive credit relief in this country.

PART II

Since Part I of this paper was written almost six months ago, much water has gone over the mill.

The Glass-Steagall Act, providing in emergency the necessary liberalization of the "cover" requirements for our Federal Reserve Notes, has become law. Through its application more than a billion dollars have been added to our "free gold" stock, which otherwise would now be less than zero.

Credit expansion through open-market purchases by the Federal Reserve Banks has become the avowed policy of our central banking system as well as of the national government. Beginning in late February, approximately 25 million dollars a week were added to Federal Reserve holdings of government securities. Moreover, after April 6 these weekly purchases were advanced to approximately 100 million dollars and were thus maintained for six successive weeks.

However, as the purchases continued on this advanced scale, not only was the entire policy subjected to more and more vicious criticism, but increasing gold outflows caused alarm in many circles. After May 18, the policy was continued only on a much reduced level. Since that date, gold exports alone have more than counteracted the further open-market purchases. Hence, for the past four weeks, the credit expansion policy may be said to have been discontinued. Indeed in the minds of superficial observers it has been tried and found wanting.

Thus was an entirely new experiment in central banking policy initiated, pursued mildly, stepped up vigorously, and finally virtually suspended. To what extent was a scientific test made? What has

been learned? In what respects was the policy found fruitful and in what others did it prove wanting?

A quantitative analysis of just what has occurred should prove illuminating.

First a dissection of Federal Reserve credit will be made. The amounts of each kind outstanding on February 24, 1932, will be compared with those of June 8, 1932, and the changes noted.

TABLE I
FEDERAL RESERVE BANK CREDIT
(in millions of dollars)

	February 24, 1932	June 8, 1932	Change
Bills discounted	835	502	−333
Bills bought	133	36	− 97
United States securities held	741	1,645	+904
Other reserve bank credit	25	16	− 9
Total reserve bank credit	1,734	2,199	+465

It will be seen that out of the very large purchases of government securities ($1,645 millions) during the period, only a comparatively small proportion ($465 millions) remained as a net accretion to Federal Reserve credit outstanding. As anticipated, a very large proportion ($333 millions) of the total was consumed through the repayment of member bank borrowings from our central banking institutions.

Further light will be thrown upon the experiment by next considering quantitatively all the various kinds of "basic credit" [1] in the United States and how they have been utilized. The amounts of each and the net changes during the period under discussion are given in Table II.

TABLE II
SOURCES AND USES OF CREDIT IN THE UNITED STATES
(in millions of dollars)

	February 24, 1932	June 8, 1932	Change
Sources of Credit			
Total reserve bank credit	1,734	2,198	+464
Monetary gold stock	4,350	3,979	−371
Treasury currency adjusted	1,780	1,796	+ 16
Total	7,864	7,973	+109
Uses of Credit			
Money in circulation	5,592	5,452	−140
Member bank reserve balances	1,878	2,112	+234
Non-member deposits, etc.	395	409	+ 14
Total	7,865	7,973	+108

[1] By basic credit is meant that upon which the banking system can in turn "base" its expansion of loans, investments, and deposits. Its items are given in Table II.

How much of the net Federal Reserve expansion, shown above to have been salvaged from the total of "open-market" purchases was in turn consumed by changes in these factors is clearly evident. On the evening of June 8, the total accretion to the credit base upon which is anchored our entire money and credit structure was but 108 million dollars. Indeed, except for cash returned from circulation during the period, there would have been an actual decline in the credit base. Only because of the release of 140 millions of hoarded cash, was the substantial increment of 234 millions saved for the use of the banking system.

What has been the effect of this addition of 234 millions to the reserve balances of member banks? The theory of the policy was that the accumulation of excess reserves in the banks would lead to an early expansion of investments and to a gradual easing of lending policies. Moreover, the theory seems well founded not only because loan and investment policies of banks are largely determined by their cash position, but also because in the past our banks have apparently rarely maintained over long periods of time reserves largely in excess of requirements.

Unfortunately, comparable data extending backward into pre-Federal Reserve days are not available for the country as a whole. For New York City, however, excess reserves [1] since 1870 are available. In Chart II they are given for all the major depression periods since that time.

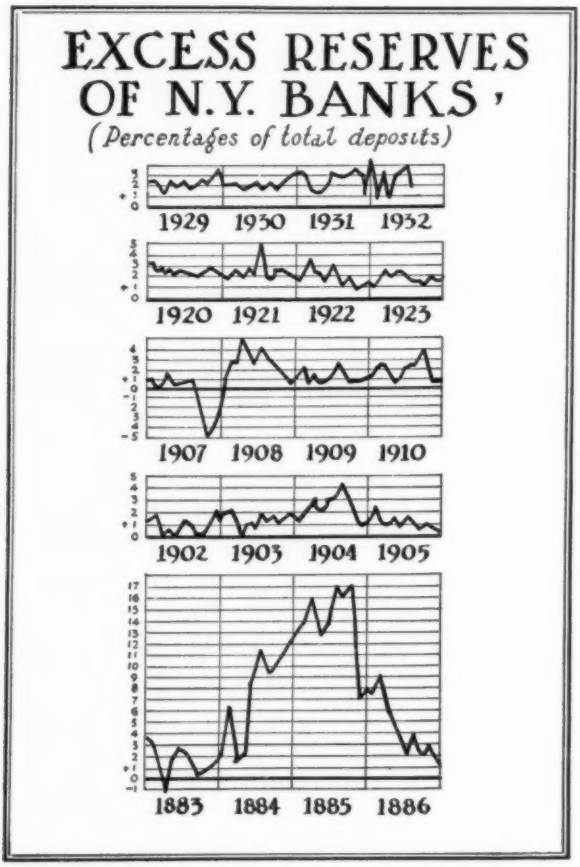
How the accumulation of such excess reserves in 1908 preceded the recovery of that year, and how similar accumulations exerted whatever weight they command on our bank officials in the months prior to the recovery from each of the major depressions of the past sixty years are compared with the recent insignificant accumulations.[2] Evidently, if the policy is to be tested, the pressure will have to be increased and maintained.

Nevertheless, certain important gains have appeared. During the period since February 24, 1932, member-bank borrowings from the Federal Reserve Banks have declined over 300 million dollars. To our banks in their present condition such a reduction in debt cannot but prove a "godsend." The accompanying decline in bank failures during the period of most rapid expansion in turn led to the release of much hoarded cash. More important still at the end of the fourth week of

[1] Prior to 1920, data are for New York Clearing House banks; for 1920 and later they are those of New York City reporting member banks. All data include cash in vault.

[2] It is recognized—especially on account of the ready purchase and sale of "Federal funds"—that New York bank excess reserves are far from satisfactory. Since pre-Federal Reserve comparisons are necessary, however, the data seem to be the best available.

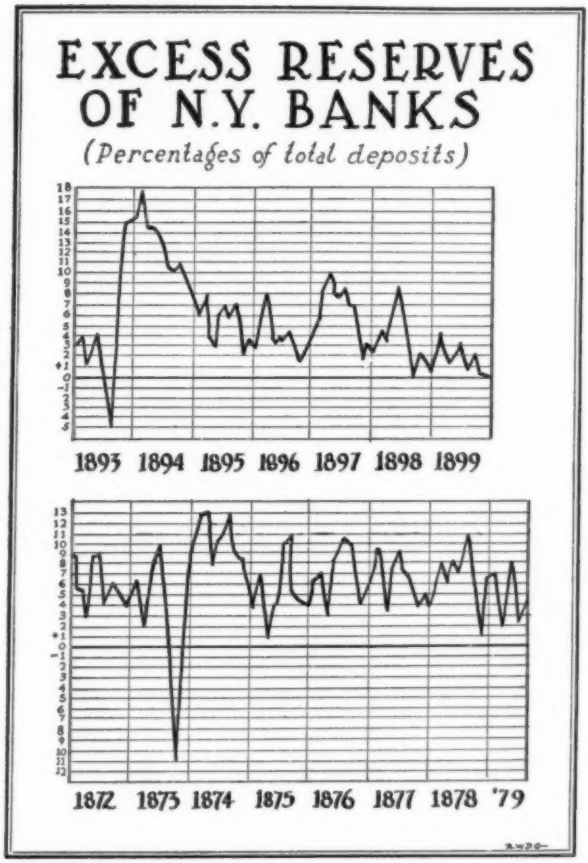
CHART II



the more vigorous policy, liquidation in the reporting member banks had ceased and had actually given place to a net expansion (of 244 million dollars in loans and investments and of 266 millions in deposits). With the big gold outflows that set in immediately afterwards, the liquidation recommenced and has continued unabated until the week ending June 15, when a slight expansion again occurred.

This slight expansion came in the face of a further small decline in member bank reserve balances. Nevertheless, the fact that excess reserves of our banking system are still considerable gives some faint hope that the long-desired turn in liquidation may finally have made its appearance. If the recent gold outflow, greatly accentuated as it was by an American capital flight, is really at an end and if Federal

CHART II (Continued)



Reserve Banks will again maintain their open-market purchases on a level to keep member bank excess reserves gradually mounting, there is some prospect that the policy may yet succeed.

It should be continually borne in mind, however, that until a bond market capable of absorbing a huge amount of refinancing and a reasonable amount of new financing has again made its appearance, the capitalist system can hardly provide the means of economic relief.

If such a bond market cannot be rebuilt by conservative credit expansion, it may soon have to be created with some one or more of the many possible unconservative inflationary measures. The alternative corrective is the continued forced liquidation of a very large amount indeed of still existing debt.

Apparently even the present much-reduced volume of indebtedness is incompatible with the low level of commodity prices. Certainly when combined with the current greatly decreased volume of business, such a price level is hopelessly inadequate. Business must be stimulated, prices raised, or a much greater portion still of our credit structure seems destined to collapse.

AN IMPROVED EQUAL-FREQUENCY MAP OF THE NORMAL CORRELATION SURFACE, USING CIRCLES INSTEAD OF ELLIPSES[1]

By Edward V. Huntington, *Harvard University*

In studying the correlation between two variables X and Y it is often important to compare a given scatter-diagram having observed values of $\bar{X}$, $\bar{Y}$, σ, τ, r, with a normal distribution having the same values of $\bar{X}$, $\bar{Y}$, σ, τ, r. (Here σ and τ denote the standard deviations of the X's and Y's respectively, and r denotes the coefficient of correlation.)

The usual method of making such a comparison is to draw the "50 per cent ellipse" on the diagram, and see whether 50 per cent of the dots lie within the ellipse. But this process is rather laborious, and becomes more so if one attempts to plot the "10 per cent ellipse," the "20 per cent ellipse," etc.

The purpose of this paper is to show how this complicated family of ellipses can be replaced by a simple family of easily plotted concentric circles. These concentric circles, together with a family of equally-spaced radial lines, will form a "cobweb map" which divides the plane into "townships of equal frequency" of any desired fineness of mesh (for example, percentiles, permilles, etc.).

This cobweb map makes possible a direct comparison between the observed distribution of dots and the theoretical distribution in the corresponding normal surface.

HOW THE MAP IS CONSTRUCTED

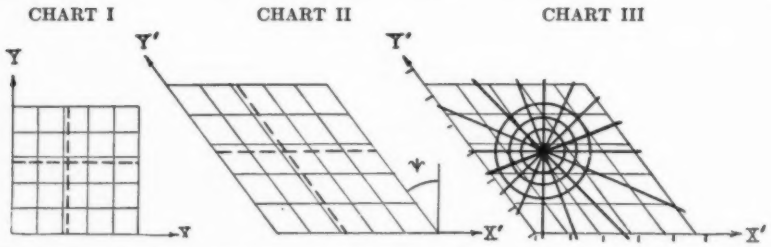
The process of constructing the map is extremely simple, as follows.

Let Chart I represent the given scatter-diagram, in which h is the class-interval for X, and k the class-interval for Y, and suppose that the means, $\bar{X}$, $\bar{Y}$ (in the original units), the standard deviations, σ, τ (in the original units), and the coefficient of correlation, r (a pure number), have been computed in the usual way. Now redraw the diagram in oblique form, as follows (see Chart II).

First, rotate the axis of Y through an angle ψ given by $\tan\psi = \dfrac{r}{\sqrt{1-r^2}}$.

Second, choose as the geometric unit of length to be used on the

[1] An abstract of this paper appeared in the *Proceedings of the National Academy of Sciences* for June, 1932.



Note 1. It will be observed that the essential feature of this process is the use of oblique coördinates. If the given diagram is re-drawn in oblique form as in Chart II, then the area around the mean-point $(\bar{X}, \bar{Y})$ can be divided into sectors of equal frequency by simply drawing a family of radial lines at equal angular intervals. A large amount of information can be obtained by counting the dots that lie within these several sectors, without drawing any of the circles. But if a table of K is at hand, much more detailed information can be secured with scarcely any additional labor, by drawing two or three of the circles. In particular, the circle with radius $\pounds(a) = \pounds(u)$ includes about 39 per cent of the total frequency (more precisely, 39.347 per cent); and the circle with radius $\pounds(a) = 3\,\pounds(u)$ includes about 99 per cent (more precisely, 98.889 per cent).

Note 2. The numerical data used in constructing Charts I, II and III, are as follows:

$h = 100$ oz., $\bar{X} = 265$ oz., $\sigma = 110.7$ oz.,
$k = 5$ dol., $\bar{Y} = 15.25$ dol., $\tau = 4.75$ dol.; $r = 0.61$.

Hence we find $\tan \psi = 0.77$; $\pounds(h) = 1.14\pounds(u)$, $\pounds(k) = 1.33\pounds(u)$; $\pounds(\bar{X}) = 3.02\pounds(u)$, $\pounds(\bar{Y}) = 4.05\pounds(u)$; $\pounds(\sigma) = \pounds(\tau) = 1.26\,\pounds(u)$. The circles shown in Chart III include 25 per cent, 50 per cent, and 75 per cent of the total frequency, respectively.

new diagram a length equal to about $\sqrt{1-r^2} \times \frac{1}{8}$ (the width of the paper), and denote it by $\pounds(u)$. (For example we may have $\pounds(u) = 1$ inch or 1 cm., on the diagram.[1]) Compute the geometric lengths $\pounds(h) = \frac{h}{\sigma} \frac{1}{\sqrt{1-r^2}}\,\pounds(u)$ and $\pounds(k) = \frac{k}{\tau} \frac{1}{\sqrt{1-r^2}}\,\pounds(u)$, to represent on the diagram the class-intervals h and k. Then, laying off successive multiples of $\pounds(h)$ and $\pounds(k)$ along the new axes of X and Y, redraw the given checker-board in the oblique form indicated in Chart II. In the new diagram we shall have $\pounds(\bar{X}) = (\bar{X}/h)\pounds(h)$ and $\pounds(\bar{Y}) = (\bar{Y}/k)\pounds(k)$; and also, incidentally, $\pounds(\sigma) = \pounds(\tau) = \pounds(u)/\sqrt{1-r^2}$.

Third, to find the circle which shall include any given percentage, p, of the total number of dots (where $p = 10$ per cent, 20 per cent, etc.), find the value of the coefficient K given by the formula $K = \sqrt{-2 \log_e(1-p)}$, and compute the geometric length $\pounds(a) = K\pounds(u)$. (A table for K as a function of p is provided.) Then using this geometric length $\pounds(a)$ as radius, and the mean point of Chart II as center, describe a circle. The circle will include the given fraction,

[1] The notation $\pounds(\sigma)$, which may be read: "*geometric* σ," is introduced in order to keep clear the distinction between a physical magnitude, such as $\sigma = 17$ ounces, and the geometric length which is used to represent this magnitude on the diagram. By the aid of this notation it is possible to adhere strictly to the useful convention that "the equality sign should never be used except between quantities of the same kind." Thus, if $\sigma = 17$ ounces, we may have $\pounds(\sigma) = 2.1$ inches on the diagram, but we could not properly write $\sigma = 2.1$ inches, since 2.1 inches is not equal to 17 ounces. [If, however, u is itself a geometric length on the diagram, then of course $\pounds(u) = u$.]

p, of the total frequency. In this way as many "frequency circles" as desired can be readily constructed.

p (per cent)	10	20	30	40	50	60	70	80	90
K	0.459	0.668	0.845	1.011	1.177	1.354	1.552	1.794	2.146

Fourth, complete the "cobweb" by drawing as many equally-spaced radial lines (see Chart III) as may be desired. (The outermost "townships" extend, of course, to infinity.)

TABLE OF K, FOR USE IN PLOTTING THE MAP OF THE NORMAL SURFACE

$$K = \sqrt{-2 \log_e (1-p)}$$

p	K	p	K	p	K	p	K	p	K
.00	.000	.20	.668	.40	1.011	.60	1.354	.80	1.794
.01	.142	.21	.687	.41	1.027	.61	1.372	.81	1.822
.02	.201	.22	.705−	.42	1.044	.62	1.391	.82	1.852
.03	.247	.23	.723	.43	1.060	.63	1.410	.83	1.883
.04	.286	.24	.741	.44	1.077	.64	1.429	.84	1.914
.05	.320	.25	.759	.45	1.093	.65	1.449	.85	1.948
.06	.352	.26	.776	.46	1.110	.66	1.469	.86	1.983
.07	.381	.27	.793	.47	1.127	.67	1.489	.87	2.020
.08	.408	.28	.811	.48	1.144	.68	1.510	.88	2.059
.09	.434	.29	.828	.49	1.161	.69	1.530	.89	2.101
.10	.459	.30	.845−	.50	1.177	.70	1.552	.90	2.146
.11	.483	.31	.861	.51	1.195−	.71	1.573	.91	2.195−
.12	.506	.32	.878	.52	1.212	.72	1.596	.92	2.248
.13	.528	.33	.895−	.53	1.229	.73	1.618	.93	2.306
.14	.549	.34	.911	.54	1.246	.74	1.641	.94	2.372
.15	.570	.35	.928	.55	1.264	.75	1.665+	.95	2.448
.16	.591	.36	.945−	.56	1.282	.76	1.689	.96	2.537
.17	.610	.37	.961	.57	1.300	.77	1.715−	.97	2.648
.18	.630	.38	.978	.58	1.318	.78	1.740	.98	2.797
.19	.649	.39	.994	.59	1.336	.79	1.767	.99	3.035−
								.999	3.717
								.9999	4.292
								.99999	4.799

Inversely, $p = 1 - e^{-\frac{1}{2}K^2}$

K	p (per cent)
0.0	0.
0.5	11.75
1.0	39.35
1.5	67.53
2.0	86.47
2.5	95.61
3.0	98.89
3.5	99.78
4.0	99.97

The mapping process here outlined would, it is believed, greatly simplify such applications of statistical theory as Dr. W. A. Shewhart, for example, has found important in Chapter IX of his recent book on *Economic Control of Quality of Manufactured Products*. Also, Professor E. B. Wilson has suggested that the device may be found useful in the

study of the random distribution of hits on a target. In general, the process provides a very expeditious graphical method by which the "normality" of any given distribution of two variables may be tested.

PROOF OF THE CORRECTNESS OF THE CONSTRUCTION

The equation of the normal surface, referred to the mean point as origin, is

$$Z = Z_o e^{-f(x,\ y)} \text{ where } f(x,\ y) = \frac{1}{2(1-r^2)}\left[\frac{x^2}{\sigma^2} + \frac{y^2}{\tau^2} - 2r\frac{xy}{\sigma\tau}\right]. \qquad (1)$$

Let $x' = \frac{x}{\sigma}\frac{1}{\sqrt{1-r^2}}$, $y' = \frac{y}{\tau}\frac{1}{\sqrt{1-r^2}}$, and $\sin\psi = r$, $\cos\psi = \frac{1}{\sqrt{1-r^2}}$.

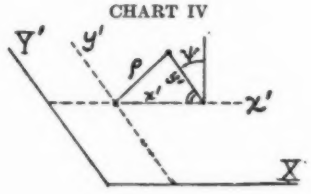
Then if x' and y' are laid off along oblique axes, as in Chart IV, the distance ρ from the origin to the point $(x',\ y')$ is given (in accordance with the law of cosines) by

$$\rho^2 = x'^2 + y'^2 - 2\cos(90° - \psi).x'y' = x'^2 + y'^2 - 2rx'y',$$

and the equation of the normal surface reduces to the simple form

$$Z = Z_o e^{-\frac{1}{2}\rho^2}. \qquad (2)$$

CHART IV



It should be noted that this process is not the same as an ordinary transformation of coördinates from rectangular to oblique axes, since the surface itself is deformed, as by shearing.

A horizontal section of this deformed surface, at height Z', will be a circle whose radius ρ' is given by $\rho'^2 = 2(\log_e Z_o - \log_e Z')$.

Let V' = the volume (or frequency) enclosed within this circle. Then V' will consist of a cylinder of altitude Z' plus a dome-shaped cap of altitude $Z_o - Z'$.

The volume of the cylinder $= \pi\rho'^2 Z' = 2\pi Z'(\log_e Z_o - \log_e Z')$.

The volume of the dome-shaped cap $= \int_{Z'}^{Z_o} \pi\rho^2 dZ = 2\pi\int_{Z'}^{Z_o} (\log_e Z_o - \log_e Z)\, dZ = 2\pi\log_e Z_o(Z_o - Z') - 2\pi(Z_o\log_e Z_o - Z_o) + 2\pi(Z'\log_e Z' - Z') = 2\pi(Z_o - Z') - 2\pi Z'(\log_e Z_o - \log_e Z')$. Hence, by adding these two parts, we have

$$V' = 2\pi(Z_o - Z'). \qquad (3)$$

But if V = the total volume under the surface, $V = \int_0^\infty Z \cdot 2\pi\rho d\rho = 2\pi Z_0 \int_0^\infty e^{-\frac{1}{2}\rho^2}\rho d\rho = 2\pi Z_0$, whence

$$\frac{V'}{V} = 1 - \frac{Z'}{Z_0}. \tag{4}$$

Therefore $V'/V = 1 - s^{-\frac{1}{2}\rho'^2}$, whence

$$\rho'^2 = -2 \log_e(1 - \frac{V'}{V}), \tag{5}$$

which was to be proved (since $\rho = V'/V$).

A COMPARATIVE STUDY OF THE INDEXES OF PRODUCTION [1]

By Y. S. Leong, *The Brookings Institution*

There are available at the present time several monthly physical volume indexes of producers' goods, of consumers' goods and of general production for the United States, but as yet no comparative study has been made of these measurements with respect to the method of construction, the component series included, the weights employed and the degree of correspondence or difference between them. The present paper attempts to supply this need.

Chart I presents a graphic comparison of the three normal-base indexes of producers' goods: the Harvard "Index of Basic Materials,"[2] Snyder's "Index of Producers' Goods,"[3] and the writer's "Index of Producers' Goods."[4] The close agreement between the movements of these measurements is remarkable. Both Snyder's and the Harvard index are made by the method of taking the weighted arithmetic means of the percentage deviations of certain physical volume series from their computed trends. In both indexes each series is corrected for seasonal variations; in the Harvard index since July, 1922 (the earliest date for which the revised index is available) and in Snyder's index, throughout the period covered, each series is reduced to an average daily basis to make allowance for Sundays and holidays before it is included in the indexes.[5] The writer's index, in contrast, is made by the aggregative method; the corrective factors for the varying number of working days

[1] The writer is indebted to the Social Science Research Council for the grant of a fellowship which made this study possible, to Dr. C. O. Hardy of the Institute of Economics and Miss Aryness Joy of the Division of Research and Statistics of the Federal Reserve Board for advice and suggestions, and to Mr. R. P. Ward of the Brookings Institution for drawing the charts.

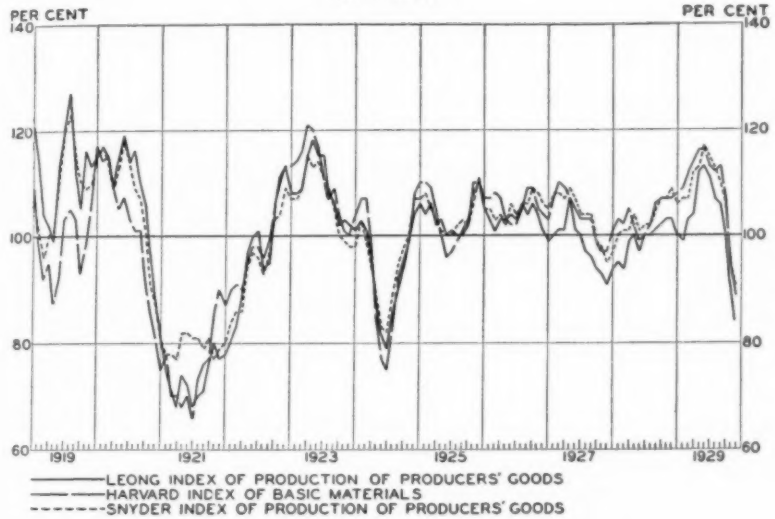
[2] E. E. Day, "Cyclical Fluctuations of the Volume of Manufacture," *Review of Economic Statistics*, January, 1923, p. 30; W. F. Maxwell, "The Revised Index of the Volume of Manufacture," *Ibid.*, May, 1929, p. 68. Other Harvard indexes discussed hereafter are also obtained from these sources.

[3] Snyder's indexes of producers' goods and of consumers' goods, which are two of the components of his Index of the Volume of Trade, have recently been revised for the third time. These indexes, which have not as yet been published, have been kindly furnished to the writer by the Reports Division of the Federal Reserve Bank of New York. See C. Snyder and L. M. Piser, "The Index of the Volume of Trade: Third Revision," this Journal, December, 1931, p. 436. For earlier versions of these indexes, see this Journal, December, 1923, p. 949, September, 1925, p. 397 and June, 1928, p. 154.

[4] Y. S. Leong, "Indexes of the Physical Volume Production of Producers' Goods and Consumers' Goods," this Journal, March, 1932, p. 21. Please refer to this source for the writer's other indexes discussed in this paper.

[5] In making a comparison between the indexes on Chart I it should be noted that the Harvard curve is composed of two sections which are joined together at July, 1922, the earlier section previous to July, 1922, is the unrevised index, the latter since July, 1922, is the revised. The old index apart from the fact that its composition and weights are different is also not corrected for non-working days.

CHART I
COMPARISON OF THE INDEXES OF PRODUCTION OF PRODUCERS' GOODS
Normal=100



are applied to and the trend and seasonal fluctuations are eliminated from the index only in its final form.

An inspection of Table I will reveal that the scopes of Snyder's and the writer's indexes are broader than that of the Harvard index. Not being represented in the fuel, lumber, non-ferrous metal and transportation equipment groups, which account for 25 and 28 per cent of the weights of Snyder's and the writer's indexes respectively, and assigning relatively little weight to the paper, leather and stone, clay and glass groups, the movement of the Harvard index is in reality largely dominated by two groups: iron and steel and textile, which represent 78 per cent of the weights assigned. The composition of Snyder's and the writer's indexes, except for the inclusion of petroleum and bituminous coal production and agricultural products in the former, is quite similar. With respect to the assignment of weights, however, Snyder's gives relatively less weight to the iron and steel and the lumber groups and relatively greater weight to the textile, fuel and the non-ferrous groups.

In view of the general agreement between the three indexes compared despite the differences in their methods of computation, their composition and their weights, it may be instructive for us to ascertain whether or not by using the same data and applying the identical weights, we may obtain results for Snyder's and for the Harvard indexes on a fixed-base, which will be equally comparable with each other and the writer's

TABLE I

COMPARISON OF THE SERIES AND OF THE RELATIVE MAGNITUDE OF THE WEIGHTS ASSIGNED TO EACH SERIES INCLUDED IN THE INDEXES OF PRODUCERS' GOODS COMPUTED BY SNYDER, THE HARVARD SOCIETY AND LEONG

Series *	Weights **		
	Snyder	Harvard Society †	Leong
Iron and steel			
Pig iron production	3.0	4.0	2.9
Steel ingots	21.0	33.8	32.2
Subtotal	(2) 24.0	(2) 37.8	(2) 35.1
Textile			
Cotton consumption	17.0	12.8	13.8
Active cotton spindle hours	....	12.9	....
Wool consumption	....	2.7	6.7
Woolen spindle activity	10.0	2.7	....
Worsted spindle activity		4.1	....
Wide, narrow, carpet and rug loom activity		5.4‡	....
Silk deliveries	7.0	....	5.6
Subtotal	(3) 34.0	(6) 40.6	(3) 26.1
Fuel			
Petroleum production	4.0	....	....
Gas and fuel oil production	....	....	.7
Lubricating oil production	....	....	.4
By-product coke production	1.0	....	2.7
Beehive coke production		....	.3
Bituminous coal production	4.0	....	....
Subtotal	(3) 9.0	....	4.1
Lumber			
Lumber production	4.0	....	12.9
Oak and maple flooring production	....	....	1.8
Subtotal	(1) 4.0	....	(2) 14.7
Nonferrous metal			
Copper production	6.0	....	3.3
Lead production	2.0	....	.9
Zinc production	1.0	....	1.3
Tin deliveries	1.0	....	.7
Subtotal	(4) 10.0	....	(4) 6.2
Stone, clay and glass			
Cement production	2.0	5.4	2.5
Plate glass	....	....	1.6
Subtotal	(1) 2.0	(1) 5.4	(2) 4.1
Paper			
Paper production other than newsprint	1.0	10.8	....
Newsprint production	2.0		.8
Wrapping paper production	....	....	1.1
Paper board production	....	....	1.5
Mechanical pulp production	1.0	....	.1
Chemical pulp production		....	.3
Paper board shipping boxes	....	....	1.0
Subtotal	(3) 4.0	(1) 10.8	(6) 4.8
Transportation equipment			
Locomotives shipped	1.0	....	.3
Freight cars shipped		....	.7
Motor trucks produced	....	....	1.1
Shipbuilding	1.0	....	.4
Subtotal	(3) 2.0	....	(4) 2.5

Leather						
Sole leather production		1.0		1.8		.7
Upper leather production—cattle		1.0		1.8		.4
Upper—calf and kid		....		} 1.8		.5
Upper—goat and kid		....		}		.8
Subtotal	(2)	2.0	(3)	5.4	(4)	2.4
Agricultural products						
Cotton movement		5.0		....		....
Wheat receipts		3.0		....		....
Corn receipts		1.0		....		....
Subtotal		9.0		....		....
Total	(25)	100.0	(13)	10.00	(29)	100.0

* The figures in parentheses indicate the number of series.
** The weights for the Harvard index are computed from data given by Maxwell, *op. cit.*, p. 72; for Snyder's, from Snyder and Piser, *op. cit.*, p. 440; for the writer's, from Leong, *op. cit.*, p. 35.
† This is known as the Index of Basic Materials.
‡ Wide Loom Activity only.

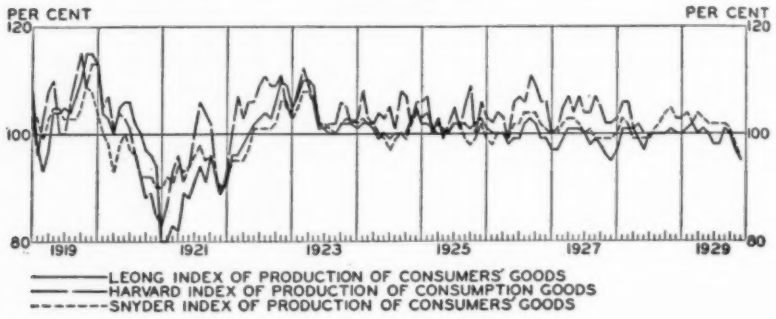
fixed-base index with respect to trend. It is clear from an examination of Table IV in which are assembled the annual indexes of producers' goods, together with those of consumers' goods and of manufactures,[1] with the average for the years 1923, 1924 and 1925 as 100, that the trends of the three indexes are dissimilar. Both Snyder's and the Harvard indexes are characterized by a more gentle slope than is the writer's. In the Harvard index, this is to be ascribed to the relatively heavy weight assigned to the textile group, which is represented on the one hand by figures for the consumption of cotton and wool, which exhibit but slight upward trends, and on the other, by machine activity data, which do not give any indication of observable trends at all. The moderate trend indicated by Snyder's index is more apparent than real. If the shipbuilding industry, which was so vastly important relative to the other industries included in the index in the early post war years, were excluded as a component, the index numbers would have become 85 for 1919, 88 for 1920 and 71 for 1921, those for the other years remaining substantially unaltered.

The three indexes of consumers' goods [2] plotted on Chart II seem to

[1] Snyder's indexes of producers' goods and of consumers' goods and the Harvard indexes of basic materials, consumption goods and manufactures, as they appear in Table IV, are constructed by the writer from annual data, by the average-of-relative method, which consists in converting the actual component series of each index into series of relatives with the averages of 1923–1925 as 100, applying the given weights to these relatives, and dividing the sums of the weighted relatives by the sum of the weights. The data used in Snyder's two indexes were supplied to the author through the courtesy of the Reports Division of the Federal Reserve Bank of New York, and the weights employed are obtained from Snyder and Piser, *op. cit.*, p. 440. The data of the Harvard indexes are obtained from various sources indicated by the authors of the indexes. (For the index of consumption goods, see Day, *op. cit.*, pp. 30–60, for the indexes of basic materials and manufactures, see Maxwell, *op. cit.*, pp. 68–109.) The weights for the index of consumption goods are those of Day (*op. cit.*, p. 54) and those for the indexes of basic materials and manufactures are the revised weights of Maxwell (*op. cit.*, p. 72).

[2] In order that the indexes may be more comparable, the writer has used the index from which automobile production has been excluded, since both the other indexes compared do not include automobile as a composite.

CHART II
COMPARISON OF THE INDEXES OF PRODUCTION OF CONSUMERS' GOODS
Normal=100



differ considerably in their movements. The Harvard index, in contrast to the smoother contour of the other two indexes, is characterized by a large number of "saw-tooth" irregularities, which are to be explained largely by the fact that the former index has not been corrected for the number of Sundays and holidays in each month. Had adjustments for non-working days been made in the Harvard index as they have been in the other two, much of the apparent dissimilarity would have disappeared and the movements between the three indexes would be more closely correlated than they now appear.

Both Snyder's and the Harvard indexes are computed by the familiar weighted averages of relatives-to-trends, while the writer's is by the aggregative method. As may be seen from Table II, the scopes of Snyder's and the writer's indexes are more comprehensive than that of the Harvard. The Harvard index represents five industries, consisting of nine series, of which four are in the food group, to which 60 per cent of the weight is assigned, and two in the tobacco group, to which 14 per cent of the weight is assigned. Inspection of the individual series indicates that all display a relatively slow rate of growth, and as reference to Table IV will show, the trend of the fixed-base composite index, computed with the same data as the normal-base, is negligible, compared to that of the writer's or that of Snyder's. Snyder's index embraces 27 series, covering eight industries, of which the most important is the food group which is assigned over 40 per cent of the total weight. Unlike either of the other two indexes, it includes as components five series of agricultural production with a weight of 14 per cent, and a series of coal production with a weight of 3 per cent. Compared to the writer's index, Snyder's assigns relatively greater weight to the chemical and textile groups and relatively less weight to the printing group. Comparison of Snyder's annual fixed-base index with that of the writ-

TABLE II

COMPARISON OF THE SERIES AND OF THE RELATIVE MAGNITUDE OF THE WEIGHTS ASSIGNED TO EACH SERIES INCLUDED IN THE INDEXES OF CONSUMERS' GOODS COMPUTED BY SNYDER, THE HARVARD SOCIETY AND LEONG

Series *	Weights **					
	Snyder		Harvard Society †		Leong	
Food						
Hogs slaughtered				10.0		9.5
Cattle slaughtered		17.0		8.0		6.0
Sheep slaughtered				....		.7
Calves slaughtered				....		.6
Wheat flour produced		7.0		29.0		7.0
Rice shipments		....		....		.3
Sugar meltings		3.0		13.0		3.9
Butter production		....		....		3.7
Cheese production		....		....		.8
Condensed and evaporated milk		....		....		1.4
Ice cream production		....		....		3.3
Corn grindings		....		....		1.6
Oleomargarine production		....		....		.4
Farm produce shipped	(5)	14.0‡		....		....
Subtotal	(11)	41.0	(4)	60.0	(13)	39.2
Textile						
Carpet and rug loom activity		....		2.0		3.2
Underwear production		1.0		....		4.9
Finished cotton goods		2.0		....		....
Hosiery		2.0		....		....
Men's and boys' suits cut		6.0		....		....
Subtotal	(4)	11.0	(1)	2.0	(2)	8.1
Enameled ware shipment		....		....	(1)	2.5
Printing						
Newsprint consumption		9.0		12.0		12.0
Book paper production		....		....		9.8
Fine paper production		....		....		5.8
Printing activity		5.0		....		....
Subtotal	(2)	14.0	(1)	12.0	(3)	27.6
Rubber						
Pneumatic tire production		6.0		....		5.2
Inner tube production		....		....		.9
Subtotal	(1)	6.0		....	(2)	6.1
Boots and shoes production	(1)	8.0	(1)	12.0	(1)	6.7
Chemical and fuel						
Petroleum products	(4)	12.0§		....		....
Gasolene production		....		....		3.4
Kerosene production		....		....		.5
Anthracite coal production		2.0		....		....
Cottonseed oil production		....		....		1.3
Subtotal	(5)	14.0		....	(3)	5.2
Tobacco						
Cigarettes				....		2.4
Cigars		6.0		11.0		1.4
Tobacco				3.0		.8
Subtotal	(3)	6.0	(2)	14.0	(3)	4.6
Total	(27)	100.0	(9)	100.0	(28)	100.0

* The figures in parentheses indicate the number of series.

** The weights for the Harvard index are computed from data given by Day, *op. cit.*, p. 54; those for Snyder's are obtained from Snyder and Piser, *op. cit.*, p. 440; and those for the writer's are from Leong, *op. cit.*, p. 36.

† This is known as the Index of Consumption Goods.

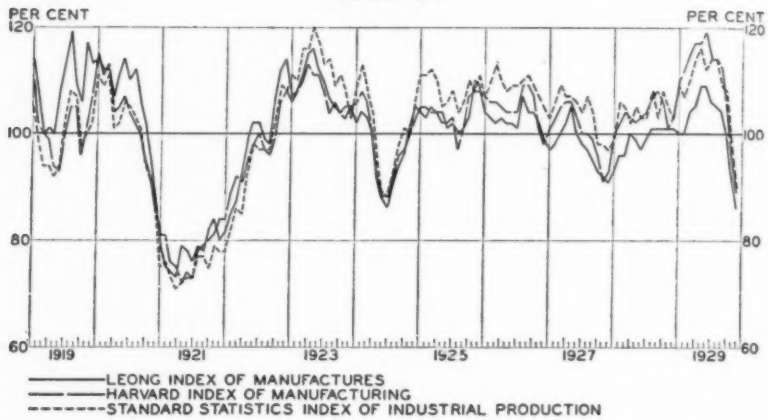
‡ The series included under "Farm Produce Shipped" in Snyder's index and their respective weights are as follows: milk receipts, 8; egg receipts, 3; potato shipment, 2; poultry receipts, 1; apple shipments, 1.

§ The series included under "Petroleum Products" in Snyder's index and their respective weights are as follows: gas and fuel oil, 1.1; kerosene, 3.9; gasolene, 9.2; lubricating oil, 10.8.

er's (Table IV) indicates that the trend of the former is more gentle—a characteristic which is to be largely explained by the fact that certain series with a more retarded rate of growth, notably the farm produce series, are given a relatively heavier weight, and certain other series, like newsprint consumption, which have a more rapid rate of growth, are weighted relatively lightly.

A comparison of the movements of the adjusted indexes of manufactures may be made by a study of Chart III. There is on the whole

CHART III
COMPARISON OF THE INDEXES OF PRODUCTION OF MANUFACTURES
Normal=100



closer correspondence between the writer's "Index of Manufactures" and the Harvard "Index of Manufacturing" than there is between the Standard "Index of Industrial Production" and either of the former two, particularly since 1922. The writer's index represents average daily production, and since July, 1922, the Harvard index is also on a similar basis. In the case of the Standard index only five [1] out of the total of 60 constituent series are corrected for non-working days, and it is because of the presence of the unadjusted components that it is marked by the more frequent occurence of the erratic month-to-month oscillations as compared with the smoother contour of the other indexes. It would have conformed more closely to the movements of the other two curves had all of its component series been put in the form of average output per working day. It will be seen that the writer's index is above the other two in 1919 and 1920 and below them since 1927. The reason for this disagreement is to be found in the difference in the slope of the respective trends. The long-time changes of the individual

[1] Pig iron, steel ingots, electric power, bituminous coal and anthracite coal production.

TABLE III

COMPARISON OF THE SERIES AND OF THE RELATIVE MAGNITUDE OF THE WEIGHTS ASSIGNED TO EACH SERIES INCLUDED IN THE INDEXES OF MANUFACTURES COMPUTED BY THE HARVARD SOCIETY, THE FEDERAL RESERVE BOARD, THE STANDARD STATISTICS COMPANY AND LEONG

Series *	Weights **							
	Harvard		F. R. Board		Standard†		Leong	
Food and kindred products								
Slaughterings and meat packing	(2)	5.5	(4)	6.2	(1)	3.0	(4)	5.5
Wheat flour	(1)	10.5	(1)	2.4	(1)	3.0	(1)	2.3
Rice cleaning		....		....		....	(1)	.1
Sugar refining	(1)	1.0	(1)	1.5	(1)	1.0	(1)	1.3
Butter, cheese and milk		....		....		....	(3)	1.9
Ice cream		....		....		....	(1)	1.1
Corn sirup, oil and starch		....		....		....	(1)	.5
Oleomargarine		....		....		....	(1)	.1
Subtotal	(4)	17.0	(6)	10.1	(3)	7.0	(13)	12.8
Textile								
Cotton goods	(2)	9.5	(1)	10.2	(2)	10.0	(1)	8.6
Woolen and worsted goods	(5)	6.5	(3)	6.8	(8)	5.0	(2)	5.2
Silk manufactures		....	(2)	3.5	(4)	3.0	(1)	3.5
Knit goods		....		....		....	(1)	1.6
Subtotal	(7)	16.0	(6)	20.5	(14)	18.0	(5)	18.9
Iron and steel								
Blast furnaces	(1)	1.5	(1)	2.2	(1)	25.0	(1)	1.8
Steel works	(1)	12.5	(1)	20.8	(1)		(1)	20.2
Plumbers' supplies		....		....	(4)	1.0	(1)	.8
Subtotal	(2)	14.0	(2)	23.0	(6)	26.0	(3)	22.8
Lumber								
Lumber		....	(1)	9.1	(1)	10.0	(1)	8.0
Flooring		....	(1)	.8‡		....	(1)	1.1
Subtotal	(0)	....	(2)	9.9	(1)	10.0	(2)	9.9
Leather and allied products								
Leather tanned, etc.	(3)	2.0	(4)	1.6	(2)	2.0	(4)	1.6
Boots and shoes	(1)	7.0	(1)	2.4	(1)	2.0	(1)	2.1
Subtotal	(4)	9.0	(5)	4.0	(3)	4.0	(5)	3.7
Rubber products								
Rubber tires and tubes	(0)	....	(2)	1.8	(1)	2.0	(2)	2.0
Paper and printing								
Wood pulp and paper	(1)	4.0	(8)	8.8	(5)	4.0	(6)	3.0
Printing and publishing	(1)	8.5	(1)	2.4		....	(3)	9.0
Subtotal	(2)	12.5	(9)	11.2	(5)	4.0	(9)	12.0
Chemicals and allied products								
Petroleum refining	(1)	6.0	(4)	1.9	(4)	2.0	(4)	2.0
Coke		....	(2)	1.9		....	(2)	1.8
Cottonseed products		....		....		....	(1)	.4
Chemicals		....		....	(7)	1.0		....
Subtotal	(1)	6.0	(6)	3.8	(11)	3.0	(7)	4.2
Stone, clay and glass products								
Cement	(1)	2.0	(1)	1.3	(1)	4.0	(1)	1.6
Glass		....	(1)	1.3		....	(1)	1.0
Clay products		....	(2)	1.3‡		....		....
Subtotal	(1)	2.0	(4)	3.9	(1)	4.0	(2)	2.6

Series *	Weights **							
	Harvard		F. R. Board		Standard †		Leong	
Metal and metal products								
Copper		....	(1)	2.2‡	(1)	4.0	(1)	2.0
Lead		....	(1)	.6	(1)	1.0	(1)	.6
Zinc		....	(1)	.7	(1)	1.0	(1)	.8
Tin		....	(1)	.5		....	(1)	.5
Subtotal	(0)	....	(4)	4.0	(3)	6.0	(4)	3.9
Tobacco manufactures								
Cigarettes, cigars and tobacco	(3)	8.0	(3)	1.1	(3)	2.0	(3)	1.5
Transportation equipment								
Motor vehicles	(1)	15.5	(1)	6.0	(2)	6.0	(2)	5.6
Railway equipment		....	(1)	.4	(3)	2.0	(2)	.7
Shipbuilding		....	(1)	.3		....	(1)	.2
Subtotal	(1)	15.5	(3)	6.7	(5)	8.0	(5)	6.5
Mineral production								
Bituminous coal		....		....	(1)	2.0		....
Anthracite coal		....		....	(1)	1.0		....
Crude petroleum		....		....	(1)	1.0		....
Subtotal	(0)	....	(0)	....	(3)	4.0	(0)	....
Electric power								
Electric power production		....		....	(1)	2.0		....
Total	(25)	100.0	(52)	100.0	(60)	100.0	(60)	100.0

* The figures in parentheses indicate the number of series.
** The weights for the Harvard index are obtained from Maxwell, *op. cit.*, p. 72; the Federal Reserve Board's are from the *Federal Reserve Bulletin*, February, 1927, p. 103; the Standard's are from the *Standard Statistics Bulletin*, Base Book Issue, 1930–1931, p. 130. Those for the author's are computed from value added data for the years 1923, 1925, and 1927 assigned to each group, industry or series.
† This is known as the Index of Industrial Production.
‡ These series have recently been omitted from the Federal Reserve Board's index.

series which compose the Harvard and the Standard indexes are determined from data, which going many years back of 1919 manifest slower rates of growth for the entire period, for which the data are available, than for the segment of the period since 1919, which is covered by the writer's index. Evidently the secular trend of the writer's index rises at a relatively more rapid rate than the composite trend (a weighted composite of the individual trends computed from the components of the index in question) of either of the other indexes compared,[1] and therefore intersects the writer's curve at points lower at the beginning and higher at the terminal of the period than it would had the rate of increase been less abrupt.

The Harvard adjusted index of manufacturing in its revised form is a composite of 25 series, representing daily average production or machine activity, computed by the method of weighted average of relatives to individual trends. This revised index, however, covers only the period since July, 1922. The curve, as shown on Chart III, is really made up

[1] The secular trend of the writer's index gives a rate of growth of 3.9 per cent per year as compared with 2.85 per cent yielded by the Standard weighted composite trend computed from 60 individual lines of growth, from which the percentage deviations composing the adjusted index are taken.

of two indexes, the original index made by Dr. Day being replaced by the revised since July, 1922. The weights of the new index are based on the average of the "net" values added in 1921, 1923 and 1925.[1] Compared to either the writer's index or that of the Standard, the revised index of the Harvard Society is more limited in scope. It is not represented in the lumber, rubber and non-ferrous metal groups; it assigns relatively greater weights to the food, leather, gasolene, tobacco and the automobile industries and relatively less weights to the iron and steel and the textile industries. As shown on Table IV, the annual fixed-base index with the average of 1923–1925 as 100, computed with the identical data and weights which enter into the adjusted index, reveals two striking disagreements: In 1921, it manifests a less severe decline and in 1929 it discloses a higher figure than that shown by any of the annual indexes of manufacture compared. The less pronounced decline in 1921 is largely due to the assignment of relatively heavy weights to those industries, such as food, leather, tobacco, etc., which showed but a mild recession as contrasted with the relatively small weights given to iron and steel, production of which fell precipitantly at this juncture. So too, the higher index number registered in 1929 finds explanation in the allocation of relatively heavy weights to the automobile and the gasolene industries, whose output rose sharply during that year.

The Standard index of industrial production is essentially an index of manufactures. It differs from the writer's index in that it includes three series of mineral production, to which is assigned a weight of 4 per cent, and one series of electric power, which is given a weight of 2 per cent. These extra series, because of the small weights allotted to them, have not modified the movements of the index from what they would have been had these components been excluded. It is computed by the method of taking the weighted arithmetic means of relatives-to-trends and its weights are derived from the "values added" and "number employed" data as reported in the 1923 Census of Manufactures. It is one of the most comprehensive indexes in current use, containing 60 series, representing practically every industrial group for which current production data are obtainable and covering by months the period since 1910.[2]

[1] The weights of the early index were based on the values added by manufactures in the census year 1919, but it should be noted that instead of being based on the "net" or actual values added, they were determined by "imputed" values added, that is, in addition to the values added of that series, the values added in the related industries or in later stages of fabrications were also included.

[2] As monthly figures for only a few series go back to 1910, it is necessary to make adjustments in the weights for the discontinued components. Instead of reducing the appropriate weights for the terminated series at the base, they are added to that for iron and steel. Thus the index registers increasingly the production of iron and steel as it goes back to 1910.

TABLE IV

COMPARISON OF ANNUAL INDEXES OF PRODUCTION
1923–1925=100

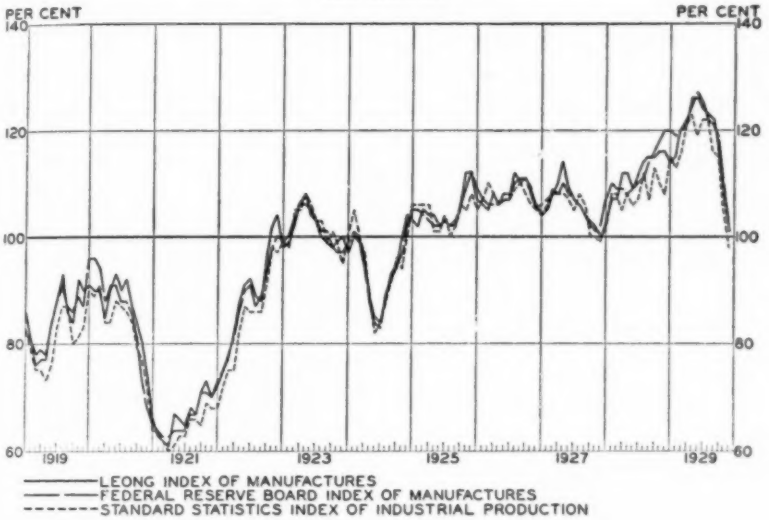
	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929
PRODUCERS' GOODS											
Harvard [1,5]	89	94	72	91	106	92	102	105	104	107	113
Snyder [1]	103	99	76	89	102	94	104	108	109	110	117
Leong	90	92	64	87	103	92	105	109	107	111	119
CONSUMERS' GOODS											
Harvard [1,7]	98	93	90	101	101	100	99	101	101	102	104
Snyder [1]	91	86	83	93	99	99	103	106	106	110	113
Leong	83	85	76	92	98	99	103	106	107	112	116
MANUFACTURES											
Harvard [1]	83	84	73	89	101	95	103	107	105	112	122
F. R. Board [2]	84	87	67	86	101	94	105	108	108	112	119
Standard [3,6]	80	84	65	84	102	94	104	107	105	108	115
Leong	84	87	67	88	101	94	105	108	106	112	118
Thomas [4]	82	85	66	86	101	94	105	108	106	112*	119*

[1] Computed by the writer. See footnote [1], *supra* p. 259.
[2] From the *Federal Reserve Bulletin*, March, 1932, p. 159.
[3] From the *Standard Statistical Bulletin*, Base Book Issue, 1930–1931, p. 126. Its original base is the value of the normal for January 1, 1923.
[4] Figures for 1919–1922 are from *Recent Economic Changes*, Vol. II, Table 27, p. 454; original base: 1899=100. Figures for other years are from Miss A. Joy's article, "Index of Production of Manufactures Derived from Census Data—1927," this JOURNAL, December, 1930, p. 457; original base: 1919=100. See text *infra* p. 268.
[5] Known as the Index of Basic Materials.
[6] Known as the Index of Industrial Production.
[7] Known as the Index of Consumption Goods.
* Preliminary.

The close correspondence between the three indexes of the production of manufactures on a fixed-base may be observed on Chart IV, on which are presented the Federal Reserve Board's "Index of Manufactures," the Standard Statistics Company's "Index of Industrial Production" and the writer's "Index of Manufactures." The similarity is closer between the writer's index and that of the Federal Reserve Board, but the Standard index would have shown greater agreement with the others had complete adjustments been made to remove the saw-tooth irregularities resulting from the varying number of working days in the different months.

The Standard index, whose base is originally the ordinate of the composite secular trend for January 1, 1923, is not independent of the adjusted index discussed in the previous paragraphs, but is a combination of the adjusted index and the weighted composite index of the computed trends of the constituent series, with the former superimposed on the latter. The computed trends are first reduced to relatives with their respective values for January 1, 1923, equal to 100, and com-

CHART IV
COMPARISON OF THE INDEXES OF PRODUCTION OF MANUFACTURES
1923–1925=100



bined in the same manner and with the identical weight factors, as are the adjusted relatives which compose the normal-base index. Where the trend of a series is absent, as in the case of activity data of textile machinery, that of a component in the same industrial group is substituted, as for instance, cotton spindle activity is represented by the trend for cotton consumption. With the values of the composite trend thus obtained, the fixed-base index may next be derived by multiplying the ordinate of the composite secular trend for the given month by the corresponding adjusted index number and dividing the product by 100.[1]

An index of manufactures on a fixed-base broader in scope and perhaps more scientifically constructed is that of the Federal Reserve Board. This is composed of 49 series,[2] covering practically all industries for which physical output data are promptly obtainable, and representing either directly or indirectly 80 per cent of the total values added by manufacture in 1923. It has not included some of the constituents of the writer's index, such, for example, as butter, cheese, manufactured milk, ice cream, etc., largely because of the tardy pub-

[1] Information concerning the Standard adjusted and unadjusted indexes is obtained from a manuscript entitled: "Standard Index of Industrial Production (Revised)," pp. 1–5, furnished to the writer through the courtesy of the Standard Statistics Company.

[2] Originally the Board's index contained 52 series, but recently three of the component series—flooring, clay products and copper—have been omitted from it.

lication of these data, which renders impracticable their incorporation in an index which is designed especially for current use. As in the writer's index, the aggregative method is utilized to combine the individual series into a composite, but it should be remarked that in the Board's index each component series is converted into figures representing daily average production for each month and adjusted for seasonal variations before it is incorporated, while in the writer's index the correction for the monthly variations in the number of working days and the adjustment for seasonal variations are made only in the computed composite index. It should also be pointed out that the weight factors of the Board's index are derived from value-added data for 1923 as reported by the Census of Manufactures, while those of the writer's index are based on the value-added figures for 1923, 1925 and 1927. The relative magnitude of the weights assigned is compared in Table III. It is interesting to note there that, while the relative magnitude of the weights assigned to the major industrial groups in the two indexes is much alike, that of the weights allocated to some of the industries and products included in them differ considerably. That the Board's index should agree so closely with the writer's is indeed remarkable in view of the differences in the basis of weighting and in the methods of eliminating the influence of the varying number of working days and of removing the seasonal variations.

The accuracy of the three monthly fixed-base indexes graphically shown on Chart IV, may now be gathered from a comparison of the annual fixed-base indexes assembled in Table IV. The most comprehensive and perhaps the most reliable measurement of the production of manufactures for the United States is the biennial index of the Bureau of the Census, constructed by Drs. Day and Thomas, which originally covered the period 1919 to 1925,[1] but which was extended to 1927 by Miss Joy of the Division of Research and Statistics, Federal Reserve Board.[2] This index represents 53 separate industries and includes 137 composite series. The odd-year figures from 1919 to 1927 of the Thomas index are obtained from this source. The intervening even-year figures have been computed by Dr. Thomas, and the figure for 1929 is a preliminary estimate. The correspondence of both the writer's index and the Board's to the Thomas index is surprisingly close. The Standard index, though comparable to the Thomas index, with respect to the rate of growth and the amplitude of the fluctuations, lies on a slightly lower level. This discrepancy is largely to be ascribed to

[1] E. E. Day and W. Thomas, *The Growth of Manufactures*, Bureau of the Census, 1928, appendices A and C, pp. 93–123, 194–196.

[2] A. Joy, "Index of Production of Manufactures Derived from Census Data—1927," this JOURNAL, December, 1930, p. 453.

the omission of the shipbuilding series and to the method of weighting the iron and steel series in the Standard index. The shipbuilding industry comprised such an important part of the manufacturing activity in the post war years, that its exclusion from the index would naturally lower the index numbers considerably for those years. In the Census index, as in the Board's and the writer's indexes, the steel ingots series is assigned a weight 10 times that of pig iron, while in the Standard index the two series are given equal weight. Because of the increasing use of scrap iron and steel in the production of steel in recent years, the growth of the production of pig iron has been less rapid than that of steel ingots. In giving pig iron and steel ingots equal weight and in allocating to the iron and steel group a weight of 26 per cent (as compared to 23 in the writer's) the Standard index has a tendency to depress the level of its index numbers, particularly in the terminal years. If the Thomas-Census index may be judged as the most accurate index of the production of manufactures, the close correspondence of the Board's, the Standard and the writer's fixed-base indexes to it, cannot but lead us to the conclusion that these monthly indexes may be trusted to give us fairly reliable measurements of the physical volume production of manufactures.

A GENERAL METHOD FOR EVALUATING MULTIPLE REGRESSION CONSTANTS

By Paul Horst

Various methods have been presented for calculating partial and multiple coefficients of correlation. Each method purports to offer certain advantages over other methods. However, none of the methods readily available in the journals have presented a general mathematical approach to the problem. The most logical and, from a mathematical point of view, the most obvious method for determining partial and multiple coefficients of correlation involving n variables is one which is based on a few elementary principles of determinants.

It is, however, quite unnecessary for the individual employing the method to be familiar with the theory involved in order to use it effectively. The method which we shall describe enables one to calculate values from which all the standard multiple regression constants may be easily derived by simple algebra. These constants which may be thus evaluated are,

1. The entire set of $\frac{n(n-1)}{2}$ partial coefficients of correlation involving the n variables.
2. All n multiple coefficients of correlation involving the n variables.
3. The entire n sets of $n-1$ regression coefficients involving the n variables.
4. The standard errors of all the regression coefficients. The standard errors of the coefficients of partial and multiple correlation are, of course, simple functions of the coefficients themselves.

To obtain the values from which all of the above constants may be derived by simple algebra requires only about fifty or seventy-five per cent more work than is involved in calculating a set of regression coefficients by the Doolittle method.

We begin with the table of zero order correlation coefficients and write these in the form of a determinant thus,

$$R = \begin{vmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{12} & 1 & r_{23} & \cdots & r_{2n} \\ r_{13} & r_{23} & 1 & \cdots & r_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{1n} & r_{2n} & r_{3n} & \cdots & 1 \end{vmatrix} \qquad (1)$$

Now we write another determinant called the adjugate determinant of R thus,

$$r' = \begin{vmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{12} & R_{22} & \cdots & R_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ R_{1n} & R_{2n} & \cdots & R_{nn} \end{vmatrix} \tag{2}$$

in which each element is the cofacter of the corresponding element in R. Since R is symmetrical, r' is also symmetrical, so that $R_{ij} = R_{ji}$.

The elements in r' provide the values from which all the statistical constants enumerated are calculated.

We let

$r_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n}$ be any partial coefficient of correlation involving all n variables;

$r_{i \cdot 1 \cdots 0 \cdots 0 \cdots n}$ be any multiple coefficient of correlation involving all n variables;

$\beta_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n}$ be any regression coefficient involving all n variables in terms of standard measures;

$\sigma_{\beta_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n}}$ be the standard error of any regression coefficient involving all n variables given in terms of standard measures.

Then, in general

$$r_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n} = \frac{-R_{ij}}{(R_{ii}R_{jj})^{1/2}} \tag{3}$$

$$r_{i \cdot 1 \cdots 0 \cdots 0 \cdots n} = \left(1 - \frac{R}{R_{ii}}\right)^{1/2} \tag{4}$$

$$\beta_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n} = \frac{-R_{ij}}{R_{ii}} \tag{5}$$

$$(N-n)\sigma^2_{\beta_{ij \cdot 12 \cdots 0 \cdots 0 \cdots n}} = \frac{R_{ii}R_{jj} - R_{ij}^2}{R_{ii}^2}.* \tag{6}$$

* Here N indicates the number of cases and n the number of variables. The equation (7) taken from Kelley has been modified in that N-n is substituted for N.

The proofs of (3), (4), and (5) are given by Kelley.[1] Kelley uses the minor notation where we use the cofactor notation. The latter notation has the advantage that the signs are taken care of in the definition of a cofactor.

The proof of formula (6) may be readily derived from Kelley's equation,[2]

$$\sigma_{b_{01 \cdot 2 \cdots n}} = \frac{\sigma_{0 \cdot 12 \cdots n}}{\sigma_{1 \cdot 2 \cdots n}\sqrt{N-n}}. \tag{7}$$

[1] T. L. Kelley, *Statistical Methods*, pp. 295 ff.

[2] *Ibid.*, p. 302, formula 283.

We substitute regression coefficients of standard measures for raw or deviation measures so that

$$\sigma_{0\cdot 12\ldots n}=\sqrt{1-r^2_{0\cdot 12\ldots n}} \tag{8}$$

and

$$\sigma_{1\cdot 2\ldots n}=\sqrt{1-r^2_{1\cdot 2\ldots n}}. \tag{9}$$

But substituting from (4), equations (8) and (9) become

$$\sigma_{0\cdot 12\ldots n}=\sqrt{\frac{R}{R_{00}}} \tag{10}$$

$$\sigma_{1\cdot 2\ldots n}=\sqrt{\frac{R_{00}}{R_{00\ 11}}}. \tag{11}$$

Substituting (10) and (11) in (7) and using β instead of b to indicate that the regressions are for standard rather than deviation measures we have

$$\sigma_{\beta_{01\cdot 2\ldots n}}=\frac{\sqrt{RR_{00\ 11}}}{\sqrt{(N-n)R_{00}}}$$

or in general

$$\sigma^2_{\beta_{ij\cdot 12\ldots 0\ldots 0\ldots n}}=\frac{R\,R_{ii\ jj}}{(N-n)R_{ii}{}^2}. \tag{12}$$

From an elementary theorem of determinants [1] we know that

$$RR_{ii\ jj}=R_{ii}R_{jj}-R_{ij}{}^2 \tag{13}$$

so that from (12) and (13) we have

$$\sigma^2_{\beta_{ij\cdot 12\ldots 0\ldots 0\ldots n}}=\frac{R_{ii}R_{jj}-R_{ij}{}^2}{(N-n)R_{ii}{}^2} \tag{14}$$

whence equation (6).

It will be noted that the right hand sides of equations (3), (4), (5) and (6) involve only the elements in r' given by (2). Our problem, then, is to solve for the elements in r'. The procedure for our solution is based on the following considerations. For illustration we shall take the case of four variables. From the elementary theorems of determinants [2] we know that

$$\left.\begin{aligned} R_{11}+r_{12}R_{12}+r_{13}R_{13}+r_{14}R_{14}&=R\\ r_{12}R_{11}+\ R_{12}+r_{23}R_{13}+r_{24}R_{14}&=0\\ r_{13}R_{11}+r_{23}R_{12}+\ R_{13}+r_{34}R_{14}&=0\\ r_{14}R_{11}+r_{24}R_{12}+r_{34}R_{13}+\ R_{14}&=0 \end{aligned}\right\} \tag{15}$$

[1] M. Bocher, *Introduction to Higher Algebra*, p. 33.
[2] T. Muir and W. H. Metzler, *Theory of Determinants*, pp. 54 and 63.

$$\left.\begin{aligned} R_{12}+r_{12}R_{22}+r_{13}R_{23}+r_{14}R_{24}&=0\\ r_{12}R_{12}+\ R_{22}+r_{23}R_{23}+r_{24}R_{24}&=R\\ r_{13}R_{12}+r_{23}R_{22}+\ R_{23}+r_{34}R_{24}&=0\\ r_{14}R_{12}+r_{24}R_{22}+r_{34}R_{23}+\ R_{24}&=0 \end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} R_{13}+r_{12}R_{23}+r_{13}R_{33}+r_{14}R_{34}&=0\\ r_{12}R_{13}+\ R_{23}+r_{23}R_{33}+r_{24}R_{34}&=0\\ r_{13}R_{13}+r_{23}R_{23}+\ R_{33}+r_{34}R_{34}&=R\\ r_{14}R_{13}+r_{24}R_{23}+r_{34}R_{33}+\ R_{34}&=0 \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} R_{14}+r_{12}R_{24}+r_{13}R_{34}+r_{14}R_{44}&=0\\ r_{12}R_{14}+\ R_{24}+r_{23}R_{34}+r_{24}R_{44}&=0\\ r_{13}R_{14}+r_{23}R_{24}+\ R_{34}+r_{34}R_{44}&=0\\ r_{14}R_{14}+r_{24}R_{24}+r_{34}R_{34}+\ R_{44}&=R \end{aligned}\right\} \tag{18}$$

A solution of equations (15) will give the values of all the elements given in the first row of the determinant on the right hand side of equation (2). Solving equations (16), (17), and (18) will give the second, third, and fourth rows respectively of this determinant. Since, however, the coefficients of all four sets of equations are the same, and since $R_{ij}=R_{ji}$, the solutions can be simplified as follows:

We reduce equations (18) by the Doolittle method. It will be remembered that the Doolittle solution gives two sets of reduced equations. The coefficients of the second set are proportional to those of the first set, being obtained by dividing each row of coefficients in the first set by the leading coefficient in that row. We write the first set of reduced equations thus,

$$\left.\begin{aligned} \gamma_{11}R_{14}+\gamma_{12}R_{24}+\gamma_{13}R_{34}+\gamma_{14}R_{44}&=0\\ \gamma_{22}R_{24}+\gamma_{23}R_{34}+\gamma_{24}R_{44}&=0\\ \gamma_{33}R_{34}+\gamma_{34}R_{44}&=0\\ \gamma_{44}R_{44}&=R \end{aligned}\right\} \tag{19}$$

For the second set of reduced equations we get

$$\left.\begin{aligned} -R_{14}+\delta_{12}R_{24}+\delta_{13}R_{34}+\delta_{14}R_{44}&=0\\ -R_{24}+\delta_{23}R_{34}+\delta_{24}R_{44}&=0\\ -R_{34}+\delta_{34}R_{44}&=0\\ -R_{44}&=-\frac{R}{\gamma_{44}} \end{aligned}\right\} \tag{20}$$

where $$\delta_{ij}=\frac{\gamma_{ij}}{-\gamma_{ii}}.$$

It may be shown[1] that

$$R=\gamma_{11}\gamma_{22}\gamma_{33}\gamma_{44} \tag{21}$$

[1] Paul Horst, "A Short Method for Solving for a Coefficient of Multiple Correlation," *The Annals of Mathematical Statistics*, February, 1932, p. 40. The notation in the present article was adopted in order to be consistent with that used in the reference cited.

hence R may be readily evaluated from the leading coefficients in (19). Transposing, (20) may be written

$$\left.\begin{aligned} R_{14} &= \delta_{12}R_{24}+\delta_{13}R_{34}+\delta_{14}R_{44} \\ R_{24} &= \delta_{23}R_{34}+\delta_{24}R_{44} \\ R_{34} &= \delta_{34}R_{44} \\ R_{44} &= \frac{R}{\gamma_{44}}. \end{aligned}\right\} \tag{22}$$

Similarly, since the coefficients in (15), (16), and (17) are the same as those in (18), these sets of equations when reduced have the same coefficients as (18). Taking the sets in reverse order, we have for equations (17)

$$\left.\begin{aligned} R_{13} &= \delta_{12}R_{23}+\delta_{13}R_{33}+\delta_{14}R_{24} \\ R_{23} &= \delta_{23}R_{33}+\delta_{24}R_{34} \\ R_{33} &= \delta_{34}R_{34}+\frac{R}{\gamma_{33}} \end{aligned}\right\} \tag{23}$$

omitting the last equation. This equation involves only R_{34} which is solved for from equations (22). Similarly, we omit the last two equations in the reduction of equations (16), writing these

$$\left.\begin{aligned} R_{12} &= \delta_{12}R_{22}+\delta_{13}R_{23}+\delta_{14}R_{24} \\ R_{22} &= \delta_{23}R_{23}+\delta_{24}R_{24}+\frac{R}{\gamma_{22}}. \end{aligned}\right\} \tag{24}$$

For equations (15) we need only the first of the reduced equations, viz.,

$$R_{11} = \delta_{12}R_{12}+\delta_{13}R_{13}+\delta_{14}R_{14}+\frac{R}{\gamma_{11}}. \tag{25}$$

Obviously, since $\gamma_{11}=1$ the δ's in (25) are merely the negatives of the r_{1i}'s, so that (25) might be written in the familiar form

$$R_{11}+r_{12}R_{12}+r_{13}R_{13}+r_{14}R_{14}=R$$

which is, of course, the expansion of the determinant by the first row. The form (25) is given merely to keep the notation uniform with that of equations (22), (23) and (24).

Equations (22), (23), (24) and (25) enable us to solve for all the elements of the determinant in equation (2). These elements provide the values from which all the statistical constants given by equations (3), (4), (5) and (6) may be calculated.

Perhaps the greatest advantage of the above outlined method is the ease with which standard errors of regression coefficients may be calculated from formula (6).

If the work sheets are systematically drawn up the solutions are

perfectly straightforward. The R_{ij}-values may be listed in a table corresponding to the determinant r' given by (2). It is a simple matter to select the required values from this table and to solve for any particular constants given by (3), (4), (5) and (6).

To illustrate the method outlined above the work sheets for a 7-variable problem are given.

Form A is merely the table of zero order correlation coefficients. These are taken as the coefficients of the normal equations. The Doolittle method must be employed for the forward solution rather than any approximation or iteration method. The variables must be arranged so that the dependent or criterion variable is the n'th variable.

The arrangement of the work sheets, however, is somewhat different than is usually described for a Doolittle reduction. This arrangement makes for convenience and economy in the calculation of the R_{ij}-values.

Form B is divided into as many sections as there are variables. In the first line of each section is copied the corresponding line from Form A. Each line in Form C gives the column summations of the corresponding section in Form B. Form C gives the coefficients of the first set of reduced equations.

Form D gives the coefficients of the second set of reduced equations, and is obtained by multiplying each line in Form C by the negative reciprocal of its leading coefficient. These negative reciprocals are written to the left of the coefficients in Form C.

It is convenient to have Forms C and D separate, since Form D gives only the values required for the back solutions. Furthermore, by matching corresponding columns of the two forms, an entire line may be calculated for the corresponding section of Form B, retaining a constant multiplier in the calculating machine.

It is best to use the values in Form D as the constant multipliers, folding the sheet after each column, as the folds are again employed in the back solutions.

A summation check column is carried as in the usual Doolittle solution.

The value at the bottom of the S-column in Form C is not a check but rather the cumulative product of all the diagonal terms on this form, and gives the value R of the determinant of the zero order correlation coefficients. (See equation (21).)

Form E gives the back solutions for the R_{ij}-values. The horizontal summations, however, instead of being carried at the right of the successive sections are recorded on Form F. Each column in Form F is a horizontal summation of the corresponding section on Form E. With the back solution work laid out in this manner an entire column may be

Form A - Zero Order Correlation Coefficients

	1	2	3	4	5	6	7
1	1.	.708	.128	.383	-.283	.412	.416
2		1.	.079	.241	-.219	.650	.250
3			1.	.078	.032	.280	.108
4				1.	-.281	-.004	.643
5					1.	.111	-.210
6						1.	.063
7							1.

Form B - Calculations for the Reduced Equations

	1	2	3	4	5	6	7	S
1	1.	+.70800	+.12800	+.38300	-.28300	+.41200	+.41600	+2.76400
2		1.	+.07900	+.24100	-.21900	+.65000	+.25000	+2.70900
	2	-.50126	-.09062	-.27116	+.20036	-.29170	-.29453	-1.96691
3			1.	+.07800	+.03200	+.28000	+.10800	+1.70500
	2		-.01638	-.04902	+.03622	-.05274	-.05325	- .35379
	3		-.00027	-.00070	-.00043	+.00835	-.00104	+ .01752
4				1.	-.28100	-.00400	+.64300	+2.06000
		2		-.14669	+.10839	-.15780	-.15933	-1.05861
		3		-.00182	-.00113	+.02167	-.00269	+ .04548
		4		-.00081	-.00195	-.00678	-.00154	- .03936
5					1.	+.11100	-.21000	+ .15000
			2		-.08009	+.11660	+.11773	+ .78221
			3		-.00070	+.01339	-.00166	+ .02811
			4		-.00467	-.01624	-.00370	- .09436
			5		-.03629	-.03034	+.09902	+ .20808
6						1.	+.06300	+2.51200
				2		-.16974	-.17139	-1.13877
				3		-.25441	+.03199	- .54031
				4		-.05645	-.01287	- .32795
				5		-.02537	+.08280	+ .17400
				6		-.04303	-.00031	- .23775
7							1.	+2.27000
					2		-.17306	-1.14982
					3		-.00398	+ .06715
					4		-.00293	- .07476
					5		-.27021	- .56783
					6		-.00000	- .00170
					7		-.00010	+ .00668

Form C - First Set of Reduced Equations or γ-Coefficients

	1	2	3	4	5	6	7	S
1	+1.00000	+ .70800	+ .12800	+ .38300	- .28300	+ .41200	+ .41600	+2.76400
2	-2.005052	+ .49874	- .01162	- .03016	- .01864	+ .35830	- .04453	+ .75209
3		-1.016931	+ .98335	+ .02828	+ .06779	+ .23561	+ .05371	+1.36873
4			-1.175530	+ .85068	- .17569	- .14691	+ .47944	+1.00751
5				-1.138627	+ .87825	+ .19441	+ .00139	+1.07404
6					-2.232142	+ .04800	- .00678	+ .44122
7						-1.819107	+ .54972	+ .54972
							$R =$	.09024

Form D - Second Set of Reduced Equations or δ-Coefficients

	1	2	3	4	5	6	7	S
1	-1.00000	- .70800	- .12800	- .38300	+ .28300	- .41200	- .41600	-2.76400
2		-1.00000	+ .02330	+ .06047	+ .03737	- .71841	+ .08928	-1.50798
3			-1.00000	- .02876	- .06894	- .23960	- .05462	-1.39190
4				-1.00000	+ .20653	+ .17270	- .56360	-1.18436
5					-1.00000	- .22136	- .00158	-1.22293
6						-1.00000	+ .01513	- .98487
7							-1.00000	-1.00000

calculated by putting the appropriate R_{ij} in the calculating machine and multiplying the corresponding column on Form D by it.

Obviously, any R_{ij} will always be calculated from a horizontal sum-

Form E - Back Solutions for the R_{ij} - Values

7	$\delta_{i2}R_{27}$	$\delta_{i3}R_{37}$	$\delta_{i4}R_{47}$	$\delta_{i5}R_{57}$	$\delta_{i6}R_{67}$	$\delta_{i7}R_{77}$	$R \div \delta_{77}$
R_{17} =	-.00503	+.00088	+.03534	-.00023	-.00102	-.06829	
	R_{27} =	-.00016	-.00558	-.00003	-.00178	+.01466	
		R_{37} =	+.00265	+.00006	-.00059	-.00897	
			R_{47} =	-.00017	+.00043	-.09252	
				R_{57} =	-.00055	-.00026	
					R_{67} =	+.00248	
						R_{77}	+.16416

6	$\delta_{i2}R_{26}$	$\delta_{i3}R_{36}$	$\delta_{i4}R_{46}$	$\delta_{i5}R_{56}$	$\delta_{i6}R_{66}$	$\delta_{i7}R_{67}$	$R \div \delta_{66}$
R_{16} =	+.10323	+.00589	-.00926	-.01262	-.08301	-.00103	
	R_{26} =	-.00107	+.00146	-.00167	-.14474	+.00022	
		R_{36} =	-.00070	+.00307	-.04827	-.00014	
			R_{46} =	-.00921	+.03479	-.00140	
				R_{56} =	-.04460	-.00000	
					R_{66} =	+.00004	+.20143

5	$\delta_{i2}R_{25}$	$\delta_{i3}R_{35}$	$\delta_{i4}R_{45}$	$\delta_{i5}R_{55}$	$\delta_{i6}R_{56}$	$\delta_{i7}R_{57}$	$R \div \delta_{55}$
R_{15} =	-.02634	-.00032	-.00614	+.03187	+.01838	+.00034	
	R_{25} =	+.00006	+.00097	+.00421	+.03204	-.00007	
		R_{35} =	-.00046	-.00776	+.01069	+.00004	
			R_{45} =	+.02326	-.00770	+.00046	
				R_{55} =	+.00987	+.00000	+.10275

4	$\delta_{i2}R_{24}$	$\delta_{i3}R_{34}$	$\delta_{i4}R_{44}$	$\delta_{i5}R_{45}$	$\delta_{i6}R_{46}$	$\delta_{i7}R_{47}$	$R \div \delta_{44}$
R_{14} =	+.01073	+.00085	-.06341	+.00453	-.00996	+.03838	
	R_{24} =	-.00015	+.01001	+.00060	-.01737	-.00824	
		R_{34} =	-.00476	-.00110	-.00579	+.00504	
			R_{44} =	+.00331	+.00418	+.05200	+.10608

3	$\delta_{i2}R_{23}$	$\delta_{i3}R_{33}$	$\delta_{i4}R_{34}$	$\delta_{i5}R_{35}$	$\delta_{i6}R_{36}$	$\delta_{i7}R_{37}$	$R \div \delta_{33}$
R_{13} =	-.02447	-.01321	+.00253	+.00071	+.01897	+.00285	
	R_{23} =	+.00240	-.00040	+.00009	+.03308	-.00061	
		R_{33} =	+.00019	-.00017	+.01103	+.00037	+.09177

2	$\delta_{i2}R_{22}$	$\delta_{i3}R_{23}$	$\delta_{i4}R_{24}$	$\delta_{i5}R_{25}$	$\delta_{i6}R_{26}$	$\delta_{i7}R_{27}$	$R \div \delta_{22}$
R_{12} =	-.20361	-.00442	+.00580	+.01053	+.06007	-.00296	
	R_{22} =	+.00081	-.00092	+.00139	+.10474	+.00063	+.18094

1	$\delta_{i2}R_{12}$	$\delta_{i3}R_{13}$	$\delta_{i4}R_{14}$	$\delta_{i5}R_{15}$	$\delta_{i6}R_{16}$	$\delta_{i7}R_{17}$	$R \div \delta_{11}$
R_{11} =	+.09529	+.00162	+.00723	+.00503	-.00132	+.01595	+.09024

Form F - Table of R_{ij}'s

	1	2	3	4	5	6	7
1	+.21404	-.13459	-.01262	-.01888	+.01779	+.00320	-.03835
2		+.28759	+.03456	-.01515	+.03721	-.14580	+.00711
3			+.10319	-.00661	+.00251	-.04604	-.00685
4				+.16557	+.01602	+.02418	-.09226
5					+.11262	-.04460	-.00081
6						+.20147	+.00248
7							+.16416

Form G - Check Sheet for the R_{ij}'s

	1	2	3	4	5	6	7	
1	+.21404	-.09529	-.00162	-.00723	-.00503	+.00132	-.01595	.09024
2		+.28759	+.00273	-.00365	-.00815	-.09477	+.00178	.09024
3			+.10319	-.00052	+.00008	-.01289	-.00074	.09023
4				+.16557	-.00450	-.00010	-.05932	.09024
5					+.11262	-.00495	+.00017	.09024
6						+.20147	+.00016	.09024
7							+.16416	.09026

mation just before it is required as a multiplier for the next column to the left.

For each section in Form E, the columns to the right of the double ruling are obtained by using those R_{ij}-values which were evaluated in the preceding section as multipliers for the corresponding columns in Form D. This is made clear by the column headings of the respective sections in Form E.

As each column in Form F is completed it is matched with the corresponding column in Form A and the products written in the corresponding column of Form G in order to check the R_{ij}-values. As each column in Form G is completed its sum plus the sum of the line to the right of its last term is evaluated and this total entered on the same line in the R-column at the right. This total should equal the cumulative product of the diagonal terms in Form C. This, of course, is the sum of the products of the zero order correlation coefficients in a given row of the determinant by their respective cofactors. The number of variables and the number of decimal places carried will determine how closely the values will check.

The method outlined gives a clean cut solution for the R_{ij}-values, and requires no re-copying of figures or unnecessary duplication of machine work. Form G provides an algebraic check on each set of back solution calculations.

For comparative purposes it may be noted that the ordinary Doolittle solution for regression coefficients carries us through Forms A, B, C, and D; and through section 7 of Form E, and column 7 of Form F.

The solution carried this far provides data for only the one multiple correlation coefficient and the $n-1$ regression coefficients. From equations (3) and (6) it is clear that it does not provide sufficient data for the calculation of any partial coefficients of correlation or standard errors of the regression coefficients.

In order to calculate all partial correlation coefficients involving the n'th variable and all standard errors of the regression coefficients involving the n'th variable, equations (3) and (6) show that it is necessary to carry the solution through the entire series of back solutions. This is necessary because all the R_{ii}-values are required in the calculation of these constants, and to get all the R_{ii}-values all the R_{ij}-values must be calculated.

However, the additional labor is little more than fifty per cent of that required for the solution of the regression coefficients alone, and when the entire table of R_{ij}-values is calculated any variable may be taken for the dependent variable, and the entire series of constants given by equations (3), (4), (5) and (6) calculated for this variable.

ADAPTATION OF NEW GEOMETRIC CODE TO MULTIPLE PUNCHING IN MECHANICAL TABULATION

By Halbert L. Dunn, *The Mayo Clinic*

The 45 and 80 column mechanical punch cards sold by both the International Business Machines Company and Powers Tabulating Machines Company, have a capacity which is too limited for many problems to which they are applied. Experience proves that secondary cards are unsatisfactory since they add to tabulating time, particularly if variables on a secondary card are being correlated with items of information on a primary card.

The geometric code which I have devised, has proved in actual use at The Mayo Clinic, to be the best method of obtaining maximal capacity of the card.

The entire cross index plan in use at The Mayo Clinic, involving all diagnostic, surgical, therapeutic and financial factors, approximately 150 items concerning each patient, has been placed on a single punch card. Each item has many variables. This number of facts would have been impossible to record, without the use, for the majority of them of a geometric code. Although this device is concerned with great detail, more than 100,000 primary cards, and more than 500,000 secondary cards can be handled in a year, and with less cost than otherwise. With use of codes which involve only a single punch per column, more than twenty cards would be required for record of the material that by use of the geometric code, can be placed on a single card.

On the punch card, as furnished, each column has ten positions, reading from above downward, from zero to 9. For use with the geometric code the two other positions, above the zero, are known as y and x. Thus, the column reads, from above downward, y, x, zero, 1, 2 and so on down to 9. This allows all combinations from 1 to 12 to be used, and if these 12 combinations are used to represent a geometric progression beginning opposite 1 at the bottom of 9 position, the positions in a given column could be represented as follows, which would be the maximal capacity of the column:

Position	*Number in the geometric progression*
y	2,048
x	1,024
0	512
1	256

GEOMETRIC CODES

NOTE.—The highest or number twelve position in the punch card column is indicated as y in the geometric code and the next highest or eleventh position as x. There are ten geometric codes each of which is indicated by the punch card position number. The lowest position number in each code is one and must be punched for all odd numbers, together with the code for the preceding even number. Punch cards using this code can be arranged in order in the following manner: (a) sort with all boxes open, (b) block out the box corresponding to the lowest position in the code used and run cards in this box again, permitting all other cards to remain as they were sorted in their respective boxes, (c) block out successively higher boxes corresponding to the next higher punch card position, each time re-running the cards from the box which has been blocked out until all cards fall in the box corresponding to the highest position used.

Position (y-9)																															
				3			2			2 3			1			1 3			1 2			1 2 3			0			3			0 2			0 2 3
				0	64	128	192	256	320	384	448	512	576	640	704	768	832	896	960	1024	1088	1152	1216	1280	1344	1408	1472	1536	1600	1664	1728	1792	1856	1920	1984
		8		2	66	130	194	258	322	386	450	514	578	642	706	770	834	898	962	1026	1090	1154	1218	1282	1346	1410	1474	1538	1602	1666	1730	1794	1858	1922	1986
		7		4	68	132	196	260	324	388	452	516	580	644	708	772	836	900	964	1028	1092	1156	1220	1284	1348	1412	1476	1540	1604	1668	1732	1796	1860	1924	1988
		8 7		6	70	134	198	262	326	390	454	518	582	646	710	774	838	902	966	1030	1094	1158	1222	1286	1350	1414	1478	1542	1606	1670	1734	1798	1862	1926	1990
			6	8	72	136	200	264	328	392	456	520	584	648	712	776	840	904	968	1032	1096	1160	1224	1288	1352	1416	1480	1544	1608	1672	1736	1800	1864	1928	1992
	8		6	10	74	138	202	266	330	394	458	522	586	650	714	778	842	906	970	1034	1098	1162	1226	1290	1354	1418	1482	1546	1610	1674	1738	1802	1866	1930	1994
		7	6	12	76	140	204	268	332	396	460	524	588	652	716	780	844	908	972	1036	1100	1164	1228	1292	1356	1420	1484	1548	1612	1676	1740	1804	1868	1932	1996
	8	7	6	14	78	142	206	270	334	398	462	526	590	654	718	782	846	910	974	1038	1102	1166	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	1870	1934	1998
			8	16	80	144	208	272	336	400	464	528	592	656	720	784	848	912	976	1040	1104	1168	1232	1296	1360	1424	1488	1552	1616	1680	1744	1808	1872	1936	2000
	8		8	18	82	146	210	274	338	402	466	530	594	658	722	786	850	914	978	1042	1106	1170	1234	1298	1362	1426	1490	1554	1618	1682	1746	1810	1874	1938	2002
		7	8	20	84	148	212	276	340	404	468	532	596	660	724	788	852	916	980	1044	1108	1172	1236	1300	1364	1428	1492	1556	1620	1684	1748	1812	1876	1940	2004
	8	7	8	22	86	150	214	278	342	406	470	534	598	662	726	790	854	918	982	1046	1110	1174	1238	1302	1366	1430	1494	1558	1622	1686	1750	1814	1878	1942	2006
		6	8	24	88	152	216	280	344	408	472	536	600	664	728	792	856	920	984	1048	1112	1176	1240	1304	1368	1432	1496	1560	1624	1688	1752	1816	1880	1944	2008
	8	6	8	26	90	154	218	282	346	410	474	538	602	666	730	794	858	922	986	1050	1114	1178	1242	1306	1370	1434	1498	1562	1626	1690	1754	1818	1882	1946	2010
		7 6	8	28	92	156	220	284	348	412	476	540	604	668	732	796	860	924	988	1052	1116	1180	1244	1308	1372	1436	1500	1564	1628	1692	1756	1820	1884	1948	2012
	8	7	6	30	94	158	222	286	350	414	478	542	606	670	734	798	862	926	990	1054	1118	1182	1246	1310	1374	1438	1502	1566	1630	1694	1758	1822	1886	1950	2014
			4	32	96	160	224	288	352	416	480	544	608	672	736	800	864	928	992	1056	1120	1184	1248	1312	1376	1440	1504	1568	1632	1696	1760	1824	1888	1952	2016
	8		4	34	98	162	226	290	354	418	482	546	610	674	738	802	866	930	994	1058	1122	1186	1250	1314	1378	1442	1506	1570	1634	1698	1762	1826	1890	1954	2018
		7	4	36	100	164	228	292	356	420	484	548	612	676	740	804	868	932	996	1060	1124	1188	1252	1316	1380	1444	1508	1572	1636	1700	1764	1828	1892	1956	2020
	8	7	4	38	102	166	230	294	358	422	486	550	614	678	742	806	870	934	998	1062	1126	1190	1254	1318	1382	1446	1510	1574	1638	1702	1766	1830	1894	1958	2022
		6	4	40	104	168	232	296	360	424	488	552	616	680	744	808	872	936	1000	1064	1128	1192	1256	1320	1384	1448	1512	1576	1640	1704	1768	1832	1896	1960	2024
	8	6	4	42	106	170	234	298	362	426	490	554	618	682	746	810	874	938	1002	1066	1130	1194	1258	1322	1386	1450	1514	1578	1642	1706	1770	1834	1898	1962	2026
		7 6	4	44	108	172	236	300	364	428	492	556	620	684	748	812	876	940	1004	1068	1132	1196	1260	1324	1388	1452	1516	1580	1644	1708	1772	1836	1900	1964	2028
	8	7	6	46	110	174	238	302	366	430	494	558	622	686	750	814	878	942	1006	1070	1134	1198	1262	1326	1390	1454	1518	1582	1646	1710	1774	1838	1902	1966	2030
			8 4	48	112	176	240	304	368	432	496	560	624	688	752	816	880	944	1008	1072	1136	1200	1264	1328	1392	1456	1520	1584	1648	1712	1776	1840	1904	1968	2032
	8		8 4	50	114	178	242	306	370	434	498	562	626	690	754	818	882	946	1010	1074	1138	1202	1266	1330	1394	1458	1522	1586	1650	1714	1778	1842	1906	1970	2034
		7	8 4	52	116	180	244	308	372	436	500	564	628	692	756	820	884	948	1012	1076	1140	1204	1268	1332	1396	1460	1524	1588	1652	1716	1780	1844	1908	1972	2036
	8	7	8 4	54	118	182	246	310	374	438	502	566	630	694	758	822	886	950	1014	1078	1142	1206	1270	1334	1398	1462	1526	1590	1654	1718	1782	1846	1910	1974	2038
		6	8 4	56	120	184	248	312	376	440	504	568	632	696	760	824	888	952	1016	1080	1144	1208	1272	1336	1400	1464	1528	1592	1656	1720	1784	1848	1912	1976	2040
	8	6	8 4	58	122	186	250	314	378	442	506	570	634	698	762	826	890	954	1018	1082	1146	1210	1274	1338	1402	1466	1530	1594	1658	1722	1786	1850	1914	1978	2042
		7 6	8 4	60	124	188	252	316	380	444	508	572	636	700	764	828	892	956	1020	1084	1148	1212	1276	1340	1404	1468	1532	1596	1660	1724	1788	1852	1916	1980	2044
	8	7	6	62	126	190	254	318	382	446	510	574	638	702	766	830	894	958	1022	1086	1150	1214	1278	1342	1406	1470	1534	1598	1662	1726	1790	1854	1918	1982	2046

| Position (y-z) | y | y | y

| Position (y–d) | | |

Position (y=7)

| | | |

2	128
3	64
4	32
5	16
6	8
7	4
8	2
9	1

All intermediate quantities, from 1 to 4,095, can be formed by addition of these quantities, and they can be formed in only one way. For example, 13 can be formed only by a punch of positions, 6, 7 and 9, which, transferred into the corresponding numbers of the geometric progression in the list just given, are equivalent to $8+4+1=13$. If every position in a column is punched the total represented equals 4,095. Obviously, the possibilities of designating different numbers, using such a device, are far in excess of those offered by other codes.

To use the geometric code, a table such as that given with this paper is necessary. This table is the geometric code and represents all the additions that can be made. To examine this code: At the bottom and furthest to the right is a group of symbols and numerals above which is the expression "Position $(y-0)$" which indicates that when one is coding data which fall within the field of application of this group, one is concerned only with column positions y to zero. In the geometric code, other groups are designated $(y-1)$, $(y-2)$, $(y-3)$ and so on up to $(y-9)$.

Above a heavy horizontal line are scattered symbols and numerals; to the left of a heavy vertical line are scattered numerals. Embraced in the right angle formed by these heavy lines are quantities of an arithmetic progression. If one were told to find a given symbol or numeral above the horizontal line and to find a given numeral to the left of the vertical line and then to read downward in the vertical line so designated and to the right in the horizontal line so designated until the lines crossed, one would find that the crossing took place on one of the quantities of the arithmetic progression. This quantity represents a fact which it is desired to code. What fact it represents could be determined, were there any occasion for it, by a list, which now will be explained, and afterward, an example will be given.

The list of data to be coded, of course, will depend on the kind of work in the office where the code is being used. Suppose the office is the statistical department of a hospital. Many diverse facts about each patient must be coded: his age, race, residence, date of admission, diagnosis, operations, a host of things even to the amount of his bill. There must be a place on the punch card for each of these facts. Facts

of a similar kind are grouped together in the list. This list is available to the coding clerk. Opposite each group in the list is printed a certain range of positions in the geometric code, $y-1$, $y-2$ and so forth, and a column number of the punch card.

Now the example: The coding clerk wishes to punch a patient's card so that it will show that he was in the hospital for eighty-three days. The list available to the clerk says that days in hospital are punched in column 29 of the punch card, and in positions in that column from zero to 9. This range, zero to 9, is included within the range y to 9 in the geometric code. Therefore, the clerk refers to that group of quantities in the geometric code which is designated $(y-9)$. The quantity 83 is an odd number and does not appear in the arithmetic progression. Therefore the clerk finds 82, which is in vertical line 3 and in horizontal line 8, 5. The greatest position in any range represents the quantity 1. The range applicable in the present example is zero to 9. Therefore, for this example 9 in column 29 represents the additional quantity, 1, which is needed to make the required quantity of 83 days. Accordingly in column 29 on the punch card the clerk punches 3, 8, 5 and 9.

To punch for diagnosis, columns 74 to 79 of the punch card are used, and an intermediate code is employed which is available to the coding clerk and is easily learned and applied.

It might be thought that this system of coding, which requires the use of intermediate steps, would be unduly difficult to understand. Nevertheless, it has not proved so. Working of three to five examples has proved sufficient to introduce the principles to new coding clerks. Certainly, it opens the way for practically inexhaustible expansion, and in the actual routine of a statistical department reduces time and effort as well as making available the gathering of facts hitherto impossible of access in a reasonable time.

ADDITIONAL ADVANTAGES OF THE GEOMETRIC CODE

The geometric code substitutes one consistent code for many. For example, if body weight could not be assigned 3 columns in a punch card, it could be punched in one column by this code instead of being grouped by intervals of 5 or 10 pounds.

It facilitates the summation of totals and averages. If the sorting machine is wired to the card counting-printer, a single run at the sorting speed of 450 a minute will permit computation of the total and the average. For example, suppose that on 1,000 punch cards money charges for services rendered or goods sold range from $1 to $3,000 punched in a certain column by this code. The result is as follows:

Punch position	Dollars for which punch position stands	Number of positions which have been punched in all the cards	Total dollars for each position
y	2,048	1	2,048
x	1,024	2	2,048
0	512	10	5,120
1	256	21	5,376
2	128	37	4,736
3	64	152	9,728
4	32	86	2,752
5	16	50	800
6	8	80	640
7	4	120	480
8	2	210	420
9	1	512	512
			34,660

The total charge is $34,660 and since there are 1,000 punch cards the average per card is $34.66. The time saving of this process is very considerable, because it involves a difference between sorting speed at 450 a minute and tabulating speed of 75 to 150 a minute. It obtains then, the advantages of capacity, coding and a faster computation of totals and averages without sacrificing the opportunity of arranging all charges in order of magnitude. The process of ordering is given in the caption of the table.

GRAPHIC PRESENTATION OF STANDARD DEVIATION

By Lewis A. Maverick

Standard deviation [1] is commonly computed arithmetically; it may, however, be computed graphically. The graphic method here presented has as its chief advantage a pictorial quality which aids to an understanding of the concept of standard deviation. Unfortunately, the graphic method is not rapid, and consequently is not suited to routine calculations. This article presents several applications of the method to time series, in which field it offers an accuracy equal to that of the conventional arithmetic procedure, and a unique adaptability to changing complexity in the time series, in that readings may be taken at varying intervals, at times selected to make possible the transcription of the details of the curve to a new chart on which measurements are to be made for the final calculations. The article also presents an application of the graphic method to frequency distributions, but in this case nothing is claimed for it beyond its illustrative character.

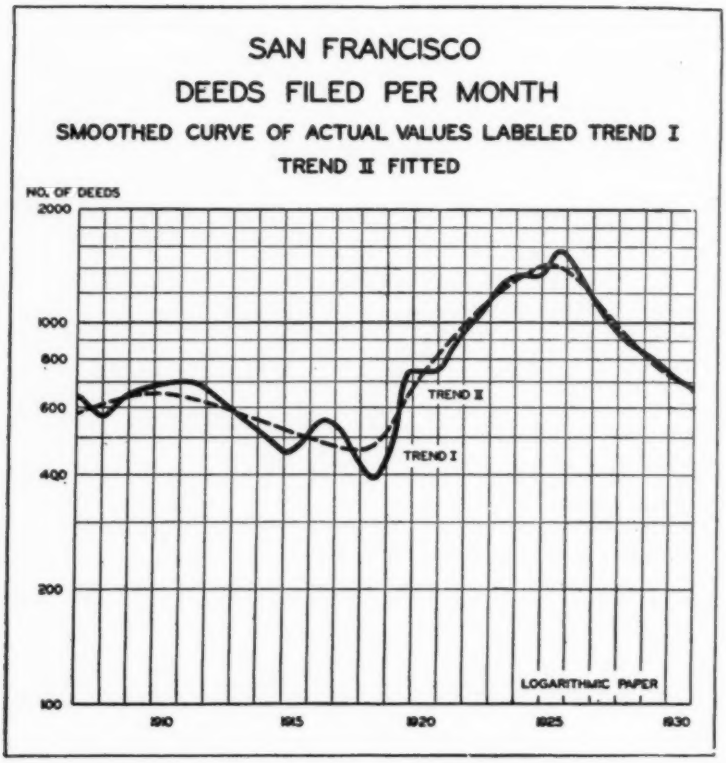
METHODS I AND II, TIME SERIES

Two graphic methods may be used to compute the standard deviation of actual values from the trend. They may be applied advantageously when the curve of actual values is not a disjointed series of straight lines connecting discrete points but a flowing curve based on a moving average or similar smoothing, and when the trend, which is, of course, a smooth curve, is the base of reference, rather than the more complex curve of normal values. Given such a chart of smoothed curve and trend, as Chart I, a second chart is constructed, on which the trend is represented by a horizontal line and the deviations or their squares plotted as ordinates from the trend as a base (Chart III, IV, or VI). The measurement of the deviations to be plotted on the second chart will be facilitated if the original chart (Chart I) is on logarithmic paper rather than arithmetic, because the properties of logarithmic paper contribute to the calculations: the ratio of actual

[1] The standard deviation, a widely used measure of the "scatter" of a set of values of some variable, is the root-mean-square deviation of the variate values from a suitable central tendency. In time series, the central tendency selected is commonly trend or normal; in frequency distributions, the arithmetic mean. Let n be the number of items observed; d the deviation of an item from the central tendency (either in actual units or in per cent); and *s.d.* the standard deviation. Then

$$s.d. = \sqrt{\frac{\Sigma d^2}{n}}.$$

CHART I



to trend is directly revealed, making unnecessary the reading of two ordinates and dividing; by the aid of a strip scale, the relative deviation of actual from trend may be read directly; or by the aid of a second scale, the square of the relative deviation may be read directly.

The deviation readings need not be made at uniform time intervals, but instead may be made infrequently where the curvature is smooth and more frequently where sharp or changing; the sole purpose of the readings is to supply sufficient detail for the construction of Chart III, IV, or VI.

Construction of the scales for reading the relative deviation and the square of the relative deviation.—The character of logarithmic paper makes it much like a slide rule, but lacking the movable scale. That scale may be supplied in the form of a strip cut from the same logarithmic paper; the scale may then be placed on the chart and multiplication and division effected "graphically." Furthermore, just as special scales may be constructed for slide-rule computation, so special scales

may be constructed for the application of particular formulas to the ratios shown on a logarithmic chart.

To read the ratio of actual to trend, use as scale a strip cut from the same logarithmic paper as the chart. Apply to the chart so that the unit point comes opposite the trend; the scale reading opposite the actual value then will be the desired ratio of actual to trend.

To read the relative deviation of actual from trend (as used in Method Ib, Chart IV; also Method II, Chart VI) the same ruling on the strip scale may be used, but with new numbers written in. At the unit or 100 point of the scale write *zero;* at 110 write $+.10$; at 150, $+.50$; at 90, $-.10$; etc. The zero point of this scale should be placed opposite the trend; then the scale reading opposite the actual value will be the desired relative deviation. (If the scale numbers be multiplied by 100, changing them to *0*, $+10$, $+50$, -10, etc., the scale will indicate percentage deviation.)

In order to construct a strip scale which will permit direct reading of the square of the relative deviation (as used in Method Ia, Chart III) a simple formula may be used.

Let d = relative deviation of actual from trend
and r = ratio of actual to trend.
Then $d = r - 1$ and $d^2 = (r-1)^2$.
Also percentage deviation = $100\,d$.

A table of values of d^2 and r may be prepared from the equation $d^2 = (r-1)^2$. Table I has been so computed and from it Chart II

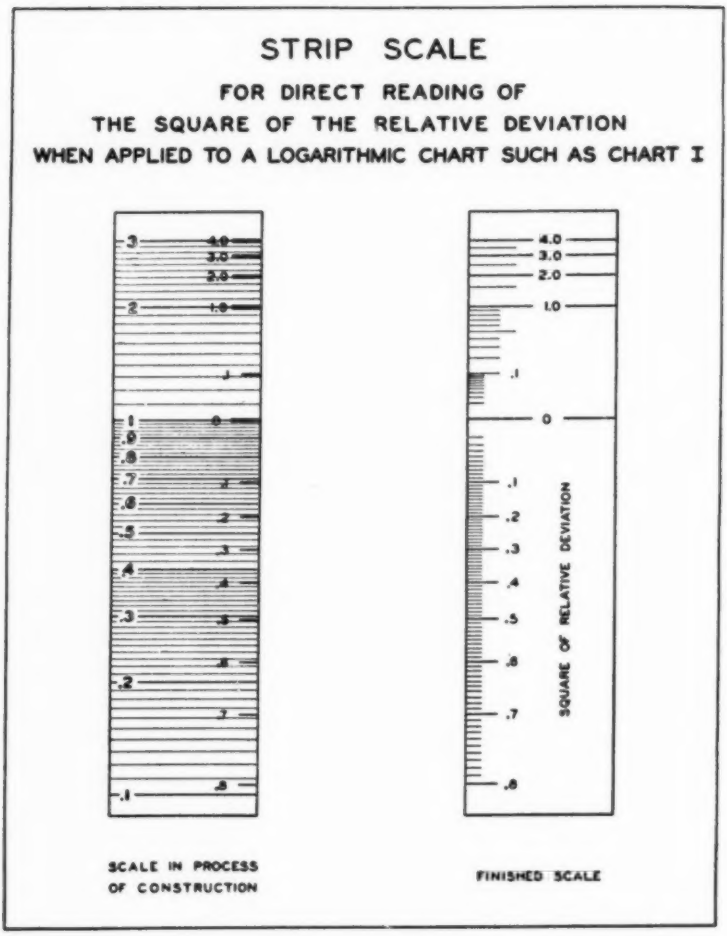
TABLE I

CORRESPONDING VALUES OF RATIO OF ACTUAL VALUE TO TREND, r, AND SQUARE OF DEVIATION FROM TREND, d^2

(To accompany Methods I and II)

Values of d^2; to be marked on scale	Corresponding values of r; to determine location of the d^2 points on the logarithmic scale
4.0	3.000
2.0	2.414
1.0	2.000
.1	1.316
.01	1.100
0.00	1.000
.01	.900
.1	.684
.2	.553
.3	.452
.4	.368
.5	.293
.6	.225
.7	.163
.8	.106

CHART II

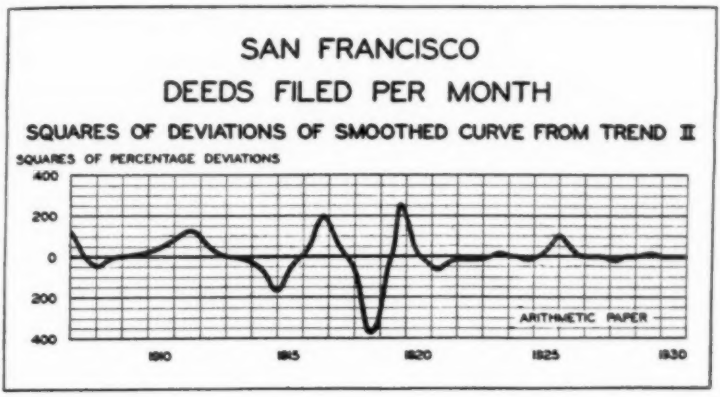

STRIP SCALE
FOR DIRECT READING OF
THE SQUARE OF THE RELATIVE DEVIATION
WHEN APPLIED TO A LOGARITHMIC CHART SUCH AS CHART I
SQUARE OF RELATIVE DEVIATION
SCALE IN PROCESS OF CONSTRUCTION
FINISHED SCALE

constructed. The points from the table were located on logarithmic paper, and then transcribed to a strip of blank paper, which is used as the scale. When applied to Chart I, this scale permits the direct reading of d^2, the square of the relative deviation of actual from trend.

METHOD I, TIME SERIES, MEASUREMENT OF AREAS

Method Ia, Squares of Deviations Plotted on Arithmetic Paper.— Method I provides two ways of securing a chart of the squares of the deviations. Under the first of these procedures (Ia), a chart of the type of Chart III is constructed: Values of the squares of the devia-

CHART III



tions are secured and plotted on a simple arithmetic chart, up or down from the base line according to the sign of the first power of the deviation. The points are connected by a smooth curve which simulates the relationships exhibited on Chart I, but distorted because of the squaring.[1]

All areas between the base line and the curve are measured by planimeter and are given positive values (the position of part of the curve below the base line is solely for the purpose of visual comparison with Chart I, and does not imply any qualification of the simple statement that the square of a negative number is positive) and the sum of the areas secured. This aggregate area, empirically determined, is Σd^2; it may be divided by the time, and the square root of the quotient taken, giving the desired quantity, the standard deviation of the smoothed time series from its trend.

Method Ib, Deviations Plotted on "Squares" Paper.—An alternative representation of squared deviations appears in Chart IV. This requires the use of specially ruled paper, which may be called "squares paper." If this method is to be used, then not the square of the deviation of the actual value from the trend, but the first power, is read from Chart I. For this purpose, as has been explained above, a strip

[1] The distortion is regular, and a knowledge of its peculiarities will help in sketching in the curve of squares of deviations. Where the curve of deviations crosses the base line at an angle, the curve of squares of deviations is tangent to the base; where the deviation has a maximum arithmetic value, the sharpness of the peak or trough is exaggerated in the curve of squares of deviations, a positive kurtosis. It would, of course, be equally accurate arithmetically and more consistent algebraically to plot the squares of all deviations on the upper or positive side of the base line; with this procedure the tangency of the curve of squares to the base line at points where the curve of deviations crosses the base line would be even clearer in its algebraic meaning. The reason this truer procedure is not elaborated in this article is that a genuine pictorial gain is effected by plotting the square of negative deviations below the base line.

CHART IV

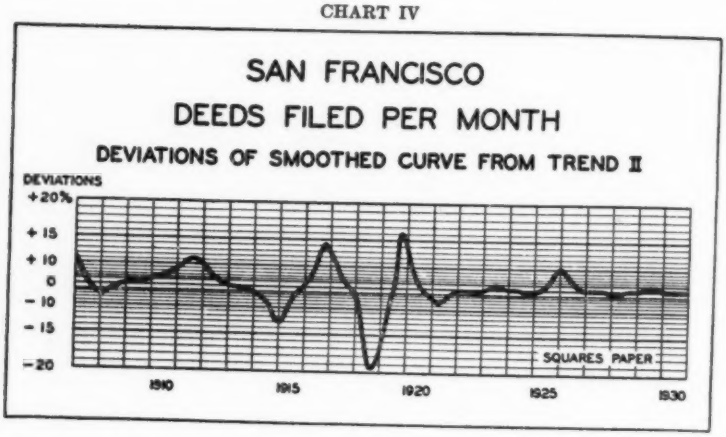
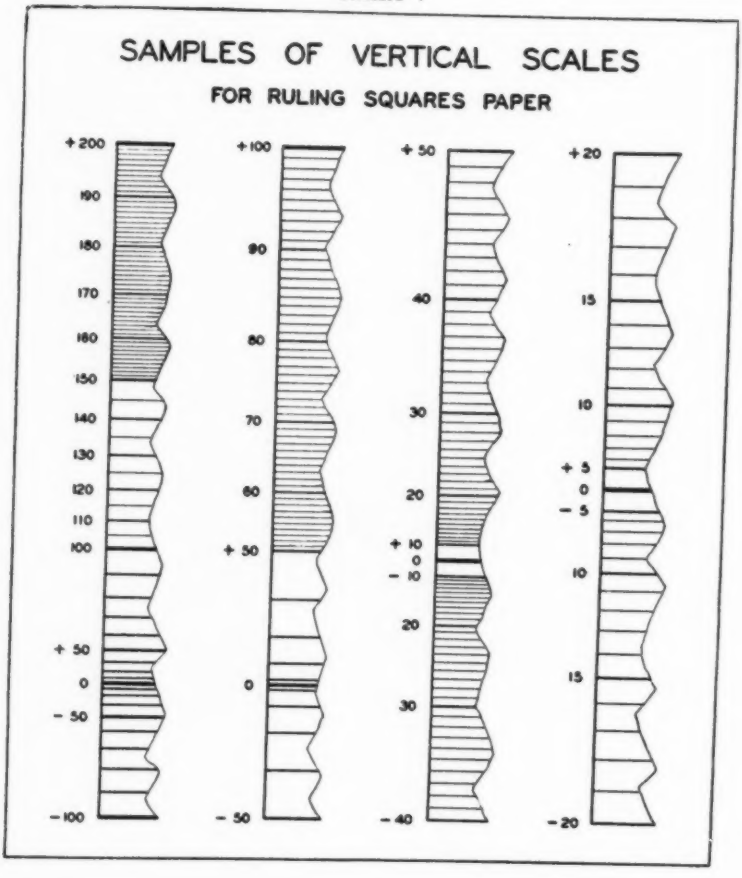


CHART V



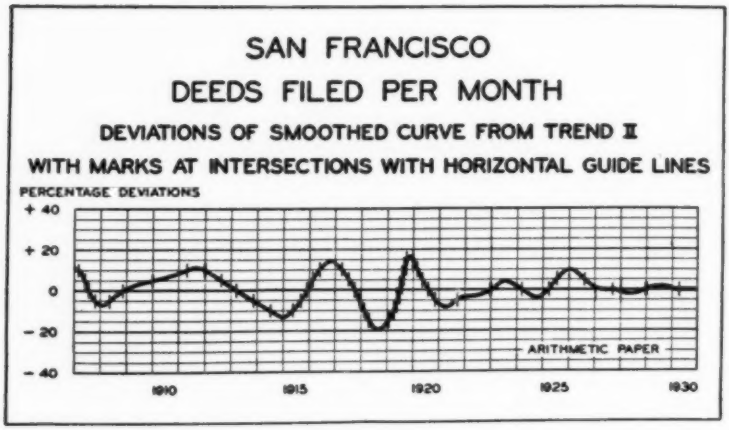
scale is used which has been cut from logarithmic paper of the same deck width as that used in Chart I. The squares paper may be ruled as in Chart IV or to suit other amplitudes of deviation, as in the samples in Chart V. The horizontal scale is arithmetic; the vertical scale is laid off from a base line near the center of the sheet in lengths proportional to the squares of the common numbers, though labeled with the first powers of those numbers. On this paper, the scale location of a deviation gives directly an ordinate whose arithmetic length is in proportion to the square rather than to the first power of the deviation. The curve obtained by plotting the first powers of the deviations on squares paper, Chart IV, is the same as that secured by plotting the squares of the deviations on arithmetic paper, Chart III. The further procedure of measuring areas and computing the standard deviation is the same as in Method Ia.

METHOD II, TIME SERIES, REDUCTION TO FREQUENCY DISTRIBUTION

In place of the first method, which involves the plotting of squared deviations and the measurement of areas by planimeter, an alternative graphic method may be employed, as illustrated in Chart VI. This method retains the advantage realized under Method I, of responding to the varying degrees of complexity or simplicity in the relationship of the curve to its trend. It reduces the time series to a weighted or frequency distribution, for which the standard deviation may be computed arithmetically. The details of the method may best be presented in a numerical example.

The relative deviations from trend may be read from Chart I and plotted up and down from a horizontal base or 100 per cent line, as has been done on Chart VI. Wherever the curve of relative deviations crosses a printed horizontal guide line, the intersection may be marked by a short vertical check, to fix clearly the date of the intersection. The element of time which will be of concern is that between two successive vertical checks. Consider, for example, the third quarter of the year 1919. Here an "element" of time is comprised between the intersection of the curve with the +10 per cent line and its intersection with the +15 per cent line. This particular element is about 1/10 year in length, and the mean ordinate, +12.5 per cent, may without significant error be taken to be the median ordinate or median deviation of the elementary section of the curve. All the elementary sections of the curve which display the same deviation, regardless of sign, are to be brought together and entered into a frequency table, in which the total time length of the elementary sections will be taken as the weight or "frequency" of the particular deviation. For accurate

CHART VI



work, however, special treatment must be given a particular class of items which defy assimilation into the frequency table, namely the points of maximum and minimum, the peaks and troughs. These elements must be handled individually, and the "representative" deviation of the element separately estimated for each.

TABLE II

DETERMINATION OF WEIGHTS TO BE ASSIGNED DEVIATIONS
(To accompany Method II)
Figures taken from Chart VI
Time measured in years from January 1, 1907
Ordinates to the intersected guide lines

(A) Time	(B) Ordinate	(C) Time Interval	(D) Mid-Ordinate	(A) Time	(B) Ordinate	(C) Time Interval	(D) Mid-Ordinate
0.2	+10			11.2	−10	.3	− 7.5
.4	+ 5	.2	+ 7.5	11.3	−15	.1	−12.5
.6	0	.2	+ 2.5	12.1	−15	.8	−17 *
.8	− 5	.2	− 2.5	12.3	−10	.2	−12.5
1.3	− 5	.5	− 6 *	12.4	− 5	.1	− 7.5
1.8	0	.5	− 2.5	12.5	0	.1	− 2.5
3.0	+ 5	1.2	+ 2.5	12.6	+ 5	.1	+ 2.5
4.3	+10	1.3	+ 7.5	12.7	+10	.1	+ 7.5
5.0	+10	.7	+11 *	12.8	+15	.1	+12.5
5.7	+ 5	.7	+ 7.5	13.0	+15	.2	+16 *
6.2	0	.5	+ 2.5	13.2	+10	.2	+12.5
6.8	− 5	.6	− 2.5	13.4	+ 5	.2	+ 7.5
7.5	−10	.7	− 7.5	13.6	0	.2	+ 2.5
8.3	−10	.8	−12 *	13.8	− 5	.2	− 2.5
8.6	− 5	.3	− 7.5	14.7	− 5	.9	− 7 *
8.9	0	.3	− 2.5	16.0	0	1.3	− 2.5
9.1	+ 5	.2	+ 2.5	17.2	0	1.2	+ 2 *
9.4	+10	.3	+ 7.5	18.2	0	1.0	− 2 *
10.3	+10	.9	+12 *	18.5	+5	.3	+ 2.5
10.5	+ 5	.2	+ 7.5	19.6	+5	1.1	+ 7.5*
10.7	0	.2	+ 2.5	20.7	0	1.1	+ 2.5
10.9	− 5	.2	− 2.5	22.0	0	1.3	− 1 *
				23.2	0	1.2	+ 1 *

* Asterisk marks maxima and minima.

Table II presents figures from Chart VI. Column A gives the time of each intersection, in years from January 1, 1907; Column B the ordinate to the intersecting guide line; Column C the length in years of each intercepted interval; and Column D the mid-ordinate or deviation of the interval. It will be noted that in Column D each peak and trough (marked with asterisk) is given a specially estimated ordinate or deviation, as in these cases the mid-ordinate between guide lines is not representative.

There follows Table III, a frequency table of the deviations from trend, in which deviations of the same arithmetic magnitude are brought together, and in which those deviations which mark the maxima and the minima, and which consequently are not readily combined, are listed individually. For the deviations of both classes, weights are assigned in accordance with the number of years that the deviations are found to obtain. In Table IV the frequency table is handled in orthodox fashion and the standard deviation computed arithmetically.

TABLE III

ASSEMBLING FROM TABLE II OF WEIGHTS TO BE ASSIGNED DEVIATIONS

(To accompany Method II)

Weights in years

Deviations + or − excluding maxima and minima			Deviations + or − for maxima and minima (the ordinate is separately estimated for each case)							
12.5%	7.5%	2.5%	1%	2%	6%	7%	11%	12%	16%	17%
Weights in years										
.6	5.5	7.4	2.5	2.2	.5	.9	.7	1.7	.2	.8

TABLE IV

CALCULATION OF STANDARD DEVIATION FROM FIGURES IN TABLE III

(To accompany Method II)

d (per cent)	(f weight in years)	d^2	fd^2	
12.5	.6	156	93.6	
7.5	5.5	56	308.0	
2.5	7.4	6	44.4	
17	.8	289	231.2	
16	.2	256	51.2	$\frac{\Sigma fd^2}{N}=\frac{1131.3}{23}=49.19$
12	1.7	144	244.8	
11	.7	121	84.7	
7	.9	49	44.1	
6	.5	36	18.0	
2	2.2	4	8.8	$\sqrt{49.19}=7.01$ per cent, say 7 per cent, the standard deviation
1	2.5	1	2.5	

$N=\Sigma f=23.0$ years $\quad \Sigma fd^2=1{,}131.3$

METHOD III, FREQUENCY DISTRIBUTIONS, THE STANDARD DEVIATION FROM THE MEAN, AREA UNDER THE OGIVE

Graphic methods may not be practicably employed for the computation of the standard deviation of a frequency distribution, but they have an illustrative quality. A simple frequency distribution is presented below, for which the standard deviation is computed arithmetically in Table V and graphically by aid of Chart VII. The arithmetic procedure needs no textual description.

On Chart VII, a horizontal base line is selected; along it the succes-

TABLE V

FREQUENCY DISTRIBUTION: COMPUTATION OF STANDARD DEVIATION FROM THE MEAN

(To accompany Method III)

Variable v		Frequency f	vf	Deviation from mean d	d^2	fd^2
Class-limits	Mid-point					
\$10.50–11.49	\$11	1	\$ 11	−3	9	9
11.50–12.49	12	3	36	−2	4	12
12.50–13.49	13	8	104	−1	1	8
13.50–14.49	14	15	210	0	0	0
14.50–15.49	15	12	180	+1	1	12
15.50–16.49	16	2	32	+2	4	8
		$n=41$	\$573			$\Sigma fd^2=49$

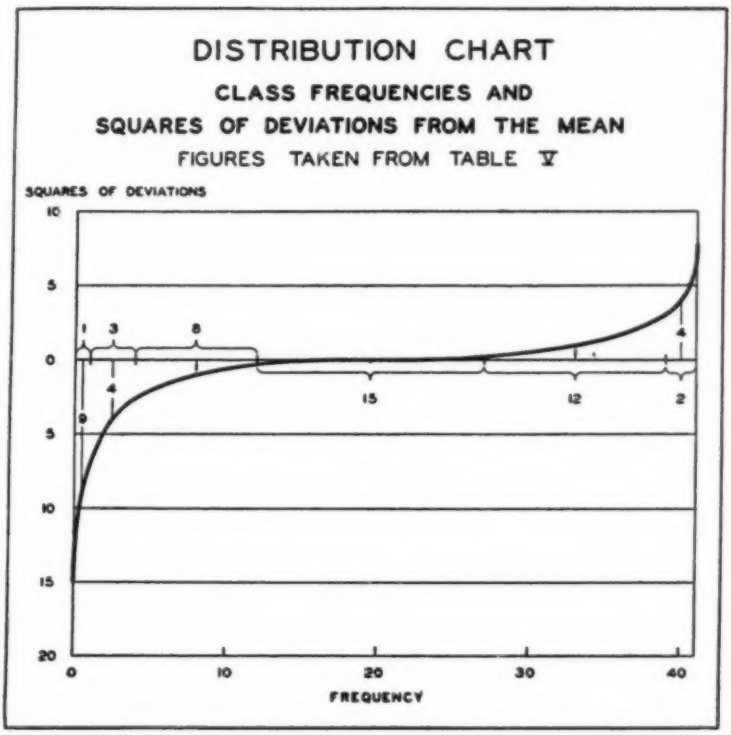
$$\text{Mean}=\frac{573}{41}=\$13.98, \text{ say } \$14.00$$

$$\frac{\Sigma fd^2}{n}=\frac{49}{41}=1.1949$$

$$s.d.=\sqrt{1.1949}=\$1.09, \text{ say } \$1.10$$

sive class frequencies are measured and the mid-point of each class located; at this central point an ordinate is erected of length proportional to the square of the corresponding deviation; the points so obtained are joined by an ogive curve which extends to the limits of the distribution. The area between the curve and the base represents the sum of the squares of the deviations from the mean. Here, as in Method I, Charts III and IV, the area below the base is positive. The total area of the figure in Chart VII is 55.7, in units which are the product of one item or frequency by the square of a deviation of \$1.00. The total area, 55.7, divided by the gross frequency, 41, gives a quotient 1.36, the mean square deviation. The square root of this number is \$1.16, the quantity sought, the standard deviation. This result may be compared with the value \$1.10, obtained by the arithmetic procedure. When the graphic method is applied to frequency dis-

CHART VII



tributions, there is regularly a bias toward a larger value of the standard deviation than by the arithmetic computation. If for the smooth ogive curve a column diagram or histogram were substituted, then the graphic presentation would be merely a replica of the arithmetic. The disparity in the results by the two methods may serve as an introduction to the question whether the items should be viewed as concentrated at the mid-point of the class or as distributed evenly through it.

NOTES

ON THE COEFFICIENT OF PART CORRELATION

By Harold D. Griffin, *Nebraska State Teachers College*

When the coefficient of multiple correlation and the multiple regression equation have been obtained by partial regression methods such as determinants, the Doolittle or iteration methods,[1] subsequent computation of a partial correlation coefficient is somewhat involved. But the calculation of an alternative concept, the coefficient of part correlation, is a simple matter when the formula is stated in terms of "beta" coefficients, as was suggested by Donald R. G. Cowan in the June number of this Journal,[2] and computed by the nomographic chart accompanying this article.

The coefficient of part correlation is a new measure developed by Bradford B. Smith and Mordecai M. B. Ezekiel.[3] Smith gives the symbol as ${}_1r_{0\cdot 23}$; while Ezekiel, whom Cowan follows, prefers the symbol ${}_{01}r_{23}$. The writer's nomographic chart follows Smith's form. This slight difference in writing the subscripts will not inconvenience users of the chart. The meaning of the coefficient of part correlation can be made clear by comparing it with the partial correlation coefficient. Partial correlation measures the concomitant variation of the dependent and one independent variable in a hypothetical universe with the influence of the other independent variables removed from both, while part correlation leaves all the variation within the independent factor and only adjusts the dependent. The coefficient of part correlation, ${}_1r_{0\cdot 23}$, or ${}_{01}r_{23}$, is literally the coefficient of correlation between

$$X_1 \text{ and } (X_0 - b_{02\cdot 13}X_2 - b_{03\cdot 12}X_3),$$

[1] For a comparative evaluation of these and other methods, see

H. D. Griffin, "On Partial Correlation vs. Partial Regression for Obtaining the Multiple Regression Equation," *Journal of Educational Psychology*, January, 1931, Vol. XXII, pp. 35–44.

H. D. Griffin, "Fundamental Formulas for the Doolittle Method Using Zero-Order Correlation Coefficients," *Annals of Mathematical Statistics*, May, 1931, Vol. II, pp. 150–153.

[2] D. R. G. Cowan, "A Note on the Coefficient of Part Correlation and of Correlation of a Dependent Variable with All but One of a Group of Other Variables," this Journal, June, 1932, Vol. XXVII, pp. 177–179.

[3] B. B. Smith, *Correlation Theory and Method Applied to Agricultural Research*, Washington, Bureau of Agricultural Research, United States Department of Agriculture, August, 1926, pp. 57–60.

M. M. B. Ezekiel, *Methods of Correlation Analysis*, New York, John Wiley & Sons, 1930, pp. 181–84, 379–80.

while the partial correlation coefficient, $r_{10\cdot 23}$, or $r_{01\cdot 23}$, is the coefficient of correlation between

$$(X_1 - b_{12\cdot 3}X_2 - b_{13\cdot 2}X_3) \text{ and } (X_0 - b_{02\cdot 3}X_2 - b_{03\cdot 2}X_3).$$

The derivation of the formula for the coefficient of part correlation is given in Ezekiel, *op. cit.*, pp. 379–380.

Smith writes the formula for the coefficient of part correlation thus

$$_1r^2_{0\cdot 23} = \frac{1}{1 + \dfrac{\sigma_0^2(1 - R^2_{0\cdot 123})}{b^2_{01\cdot 23}\sigma_1^2}}.$$

Ezekiel prefers the formula

$$_{01}r^2_{23} = \frac{b^2_{01\cdot 23}\,\sigma_1^2}{b^2_{01\cdot 23}\,\sigma_1^2 + \sigma_0^2(1 - R^2_{0\cdot 123})}.$$

Cowan used Ezekiel's form in constructing the formula which he suggests, for as $\beta_{01\cdot 23} = b_{01\cdot 23}\sigma_1/\sigma_2$, so

$$_{01}r^2_{23} = \frac{\beta^2_{01\cdot 23}}{\beta^2_{01\cdot 23} + (1 - R_{0\cdot 123})}.$$

The present writer working on the problem in the fall of 1929, prior to the publication of Ezekiel's book, constructed his nomogram on Smith's version. Therefore, the formulas on the accompanying chart are written as

$$_1r_{0\cdot 23} = \sqrt{\frac{1}{1 + \dfrac{1 - R^2_{0\cdot 123}}{\beta^2_{01\cdot 23}}}} \text{ and}$$

$$_1r^2_{0\cdot 23} = \frac{1}{1 + \dfrac{1 - R^2_{0\cdot 123}}{\beta^2_{01\cdot 23}}},$$

which may be generalized as follows:

$$_1r_{0.^2 23\ \ldots\ n} = \frac{1}{1 + \dfrac{1 - R_{0^2\cdot 123\ \ldots\ n}}{\beta^2_{01\cdot 23\ \ldots\ n}}} = {}_{01}r^2_{23\ \ldots\ n}$$

$$_2r_{0.^2 13\ \ldots\ n} = \frac{1}{1 + \dfrac{1 - R_{0.^2 123\ \ldots\ n}}{\beta^2_{02\cdot 13\ \ldots\ n}}} = {}_{02}r^2_{13\ \ldots\ n}$$

$$_3r_{0.^2 12\ \ldots\ n} = \frac{1}{1 + \dfrac{1 - R_{0.^2 123\ \ldots\ n}}{\beta^2_{03\cdot 12\ \ldots\ n}}} = {}_{03}r^2_{12\ \ldots\ n}$$

etc.

CHART I

Given the multiple correlation coefficient, R, and the partial regression coefficient, β, to find the coefficient of part correlation, according to the formulas:

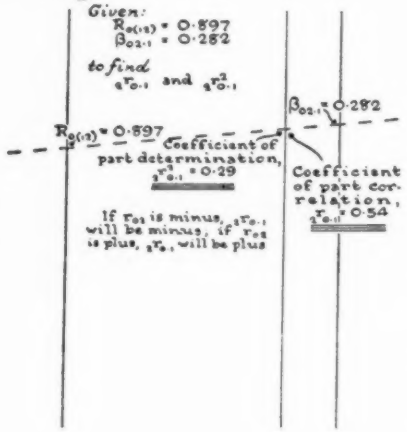
$$ {}_1r_{0\cdot23} = \sqrt{\frac{1}{1+\frac{1-R^2_{o(123)}}{\beta^2_{01\cdot23}}}} $$

$$ {}_2r_{0\cdot13} = \sqrt{\frac{1}{1+\frac{1-R^2_{o(123)}}{\beta^2_{02\cdot13}}}} $$

$$ {}_3r_{0\cdot12} = \sqrt{\frac{1}{1+\frac{1-R^2_{o(123)}}{\beta^2_{03\cdot12}}}} $$

The coefficient of part determination, ${}_1r^2_{0\cdot23}$, is also given for values of R and β.

Key:


Given:
$R_{o(12)}$ = 0·897
$\beta_{02\cdot1}$ = 0·282
to find
${}_1r_{0\cdot1}$ and ${}_1r^2_{0\cdot1}$
$R_{o(12)}$ = 0·897
$\beta_{02\cdot1}$ = 0·282
Coefficient of part determination, ${}_1r^2_{0\cdot1}$ = 0·29
Coefficient of part correlation, ${}_1r_{0\cdot1}$ = 0·54
If r_{01} is minus, ${}_1r_{0\cdot1}$ will be minus, if r_{01} is plus, ${}_1r_{0\cdot1}$ will be plus

hgd
(32)

The first subscript to the right of r in the formulas on the chart designates the dependent variable (or criterion), the subscripts following the period denote the independent variables whose functions have been subtracted from the dependent variable, and the subscript to the left of r is the remaining independent which in itself is unadjusted but is to be correlated with the adjusted dependent.

The squared form of a coefficient of part correlation may be considered as a coefficient of part determination. Thus, ${}_{01}r^2_{23}$ or ${}_{1}r^2_{0\cdot 23}$ may be explained as measuring the per cent variance attributable to X_1 which remains in X_0 after the net effects of the other variables are taken into account.[1]

This nomographic chart will compute the coefficients of part correlation and the coefficients of part determination in problems of any number of variables. The writer suggests that a straight line scratched by a needle on the under-surface of a strip of celluloid makes the most satisfactory straight-edge to use with it.

[1] Ezekiel, *op. cit.*, p. 183.

CENSUS OF DISTRIBUTION [1]

By Robert J. McFall

The new Census of Distribution, taken as a part of the Fifteenth Decennial Census, is essentially a census of the merchandising trades.

In covering the retail and wholesale trades, information was gathered on the organization and operating expense of each business establishment covered, the net sales and stocks.

The principal inquiries on the schedules were as follows:

Description of establishment
Description of business
Proprietors and firm members
Number of employees, and salaries and wages
Rent paid
Interest paid
All other expenses
Stocks on hand for sale
Net sales of merchandise and products
Credit sales
Commodity sales

In addition, the various schedules carried special inquiries relative to the field covered, such as repair and service operations, sales of cream and milk, slaughter of meat animals, etc.

An establishment-to-establishment canvass was made by special corps of census-takers in the larger cities and by the regular enumerators in places of less than 10,000 inhabitants.

For the retail trade there were four main schedules—viz., one for food stores, another for drug stores, the third for other stores and a fourth which was a short form on which all small retail establishments reported. For the wholesale trade there were two schedules—viz., one for food dealers, and the other for other wholesale dealers of all kinds.

A combined schedule for both retail and wholesale was used in covering the automotive trades. In the small towns and rural communities, an omnibus schedule was used which covered both retail and wholesale establishments.

As a matter of organization the census of construction was taken as a branch of the Census of Distribution.

[1] Substituted for the usual Progress of the Census report by Dr. J. A. Hill.

For census purposes, the term "retailer" includes all establishments selling goods at retail direct to consumers. Thus restaurants are included, for their sales are to the general public as consumers.

Stores selling feed and supplies to farmers, and selling hardware to builders, are also included, because they conduct their businesses in the manner of retailers and are generally considered to be such. All stores are classified in accordance with the description or designation commonly used by the trade and by the public-grocery stores, drug stores, etc.

A wholesale establishment, for purposes of the census, includes the entire non-retail field among the merchandising concerns. Hence wholesale trade, as defined for census purposes, embraces all establishments engaged in the purchase and sale, or distribution of goods on a wholesale basis. In addition to wholesalers of the conventional type, the census covers all establishments performing the wholesale function in business. It includes assemblers and country buyers of farm products, such as elevators, produce buyers, coöperative marketing associations, etc.; likewise brokers, commission merchants, selling agents, etc.

In order to make the information available as quickly as possible, the statistics for the retail and wholesale trades were first given out in preliminary form for cities; the retail reports being issued for all cities of 10,000 population and more, the wholesale for cities of 100,000 and more. Preliminary figures for states and other divisions, by kind of business, were also made available in United States Summaries for these trades.

Complete and final statistics are now being issued in reports for states. These reports have now been issued for many states, and the others are in the process of publication; and will all be available within a short while.

The Retail Reports.—The items of information include number of stores; the number of employees, distinguishing full time and part time; total pay roll, distinguishing payments to full time employees and to part time; the total net sales; value of stock on hand at the end of the year; number of proprietors and firm members not on pay roll; expenses, showing separately total pay roll, wage value of proprietors' services, and rent paid for leased premises, but not including the cost of merchandise sold.

The number of stores and annual net sales are classified by kind of store, distinguishing grocery stores, furniture stores, dry goods stores, etc.; also by type of operation, distinguishing single store independents, two and three store independents, local chains, sectional chains, and national chains.

There is also a classification by size of business, distinguishing stores with sales from $100,000 to $199,999, from $50,000 to $99,999, etc., also a classification with reference to the percentage of business done on credit, distinguishing all-cash stores, stores doing from 1 to 10 per cent of sales on credit, from those doing from 11 to 20 per cent credit, and so on.

The reports include an analysis or breakdown of sales by commodities so far as the data could be obtained. The commodity coverage, as it is called, i.e. the percentage which the sales reported by commodities constitute of the total sales, varies widely in different kinds of stores. In the case of department stores, for instance, frequently 100 per cent of the sales were reported by commodities, whereas in other kinds of stores the commodity coverage might in some cases be much smaller.

Tables have been included in the reports showing what percentage the sales of any given commodity form of the total sales reported by commodities; and even where the commodity coverage is small it is believed that these percentages have significance and can be used as a basis for estimating the total sales of that commodity in the given class of stores. The Bureau has published a bulletin on the uses that can be made of the commodity data as here presented.

The number of stores and net sales by principal kinds of business are shown separately for each county and for each city or incorporated place of 1,000 population or over. Fuller detail is given for individual cities of 30,000 population or more.

The following is a list of the tables shown for the state as a whole:

Retail distribution, by kinds of business
Operating expenses—all retail stores
Seasonal employment characteristics
Sales by size of business
Retail distribution by types of operation
Retail distribution—17 kinds of business by types of operation
Sales classified according to degree of credit
Credit business
Receipts other than from the sale of merchandise
Merchandise manufactured, sales to other retailers, returned goods, and other related data
Legal forms of organization
Retail distribution by counties (including cities and towns of 1,000 population and over)
County distribution by kinds of business
Sales by commodities

The Wholesale Reports.—The items of information include number of

establishments; number of employees; total salaries and wages; total expenses, including salaries and wages; cost value of stocks on hand at end of year; net sales, amount and per cent of total; credit sales, amount and per cent of net sales; sales at retail to ultimate consumers; sales to industrial consumers. These statistics are given by kind of business (automotive, hardware, dry goods and apparel, electrical, jewelry and optical goods, etc.); and by type of establishment (full-service and limited-service wholesalers, bulk tank stations, chain-store warehouses, district sales offices, etc.).

The number of establishments, number of employees, salaries and wages, total expenses and net sales are also given by character of organization, distinguishing individual proprietorships, partnerships, corporations, and coöperative associations.

The numbers of marketing units (single-unit, two-unit, up to 100 or more) are likewise shown, with the number of establishments, their net sales and total expenses (including salaries and wages) given by unit groups.

Employees engaged are distinguished as salesmen, executives, and all other employees.

Wholesale merchants proper, as distinguished from brokers, commission merchants, etc., are classified according to annual volume of net sales, and by kinds of business. Commodity sales and commodity coverage are shown in considerable detail.

Information in full detail is given for the state as a whole and for cities of 500,000 population and over and in less detail for smaller cities.

The following is a list of the tables shown for the state as a whole:

Summary of wholesale trade by principal kind of business classifications
Wholesalers only, by kinds of business
Wholesale trade, by kind of business
Wholesale trade, by type of establishment
Number of establishments, by type and kind of business
Net sales, by type of wholesale establishment and kind of business
Total expenses, by type of wholesale establishment and kind of business
Summary of wholesale trade, by character of organization
Wholesale trade, by the number of marketing units and kind of business
Employees engaged in wholesale trade, by kinds of business
Classification of wholesale merchants according to net sales, by kind of business

Number of establishments, by kind of business

Commodity sales, by kind of business and type of establishment.

In addition to the state reports, the Distribution Division is issuing special and trade studies covering the principal phases of the census. Some of these reports are already available, and the others are either in the process of printing or preparation.

The retail and wholesale reports for states, together with United States Summaries for each trade, are to be assembled in the final quarto volumes of the Census of Distribution. These volumes will contain all the basic statistics on these trades, by geographic divisions of the United States.

A METHOD OF CALCULATING WEEKLY SEASONAL INDEXES

By Leroy M. Piser, *Federal Reserve Bank of New York*

A growing realization that monthly data do not give a sufficiently current portrayal of the changing economic situation has led to an increasing interest in weekly data. For the satisfactory interpretation of such figures it is often necessary to adjust for seasonal variations. The weekly measurement of seasonal fluctuations, however, involves two important difficulties. First, as weekly data are available for relatively few series and generally over only a short period of time, the weekly indexes should be considered as supplementary to the monthly indexes and should accordingly check with them. Second, it is necessary in a number of series that the seasonal index make allowance for the variations from year to year in the date of each week. In the case of the amount of reserve bank credit outstanding, for example, there is a measurable difference whether the second week of the year ends on

TABLE I
CALCULATION OF RATIOS FOR WEEKLY SEASONAL INDEX ON COMBINED AMOUNT OF RESERVE BANK CREDIT AND MONETARY GOLD STOCK IN THE UNITED STATES

Week ended	(1) Average daily data in millions of dollars	(2) Distribution of monthly adjusted data	(3) Ratio column (1)÷ column (2)
1922			
January 7	5,168	4,983*	103.7
14	5,004	4,983*	100.4
21	4,944	4,983**	99.2
28	4,914	4,988	98.5
February 4	4,941	4,993	99.0
11	4,958	4,997	99.2
18	4,932	5,002**	98.6
25	4,903	5,002	98.0
March 4	4,951	5,003	99.0
11	4,930	5,003	98.5
18	4,959	5,003**	99.1
25	4,923	5,004	98.4
April 1	4,952	5,005	98.9
8	4,966	5,005	99.2
15	4,985	5,006**	99.6
22	4,970	5,014	99.1
29	4,942	5,022	98.4
May 6	5,002	5,031	99.4
13	4,989	5,039	99.0
20	4,970	5,047**	98.5
27	4,955	5,041	98.3
June 3	4,980	5,035	98.9
10	4,976	5,029	98.9
17	4,988	5,023**	99.3
24	4,925	5,022†	98.1

* Calculated partly from adjusted figure for December, 1921, which is not shown.
** Monthly adjusted figure.
† Calculated partly from adjusted figure for July, 1922, which is not shown.

January 8 or on January 9, and the difference between January 8 and January 11 amounts to nearly 20 per cent of the amplitude of the seasonal index.

The method of computing weekly seasonal indexes which has been used at this bank for a number of years obviates both of these difficulties. The weekly index is derived from the monthly index and consequently checks exactly with it. Moreover, the method may be used to obtain an index either with or without allowance for the varying date of each week.

The accompanying tables illustrate the steps essential to the calculation of a weekly seasonal index on the combined amount of reserve bank credit and monetary gold stock in this country. After the monthly data have been corrected for monthly seasonal variations by a standard method, each adjusted figure is placed opposite the week

TABLE II

CALCULATION OF WEEKLY SEASONAL INDEX ON COMBINED AMOUNT OF RESERVE BANK CREDIT AND MONETARY GOLD STOCK IN THE UNITED STATES

Week ended		(1) Ratios computed in Table I	(2) Average of daily ratios	(3) Moving median of column (2)	(4) Index
January	*Year*				
1	1927	104.9	104.9	104.8*	104.9
2	1926	104.8	104.8	104.8*	104.8
3	1925	104.6	104.6	104.6	104.7
4	1930	104.5	104.5	104.5	104.5
5	1924	104.1	104.2	104.2	104.2
	1929	104.2			
6	1923	103.7	103.7	103.8	103.8
7	1922	103.7	103.8	103.7	103.6
	1928	103.9			
8	1927	103.2	103.2	103.2	103.2
9	1926	102.5	102.5	102.5	102.5
10	1925	102.4	102.4	102.4	102.4
11	1930	101.5	101.5	101.7	101.7
12	1924	101.1	101.2	101.5	101.5
	1929	101.3			
13	1923	101.7	101.7	101.2	101.2
14	1922	100.4	100.7	100.7	100.7
	1928	100.9			
15	1927	100.4	100.4	100.4	100.4
16	1926	100.1	100.1	100.1	100.2
17	1925	100.1	100.1	100.1	100.0
18	1930	99.7	99.7	99.7	99.7
19	1924	99.5	99.7	99.7**	99.6
	1929	99.9			
20	1923	99.4	99.4	99.4**	99.5

* Calculated partly from December data, which are not shown.
** Calculated partly from later January data, which are not shown.

which includes the fifteenth of that month. This is illustrated in Table I, column 2.[1] A weekly series without seasonal influence is obtained for the intervening weeks of each month by straight-line interpolation between the monthly adjusted data. The ratio between each actual weekly figure and the corresponding adjusted figure is calculated in

[1] Had the monthly deseasonalized data shown marked irregularity their fluctuations would have been reduced by the computation of a moving average.

column 3; these ratios reflect the seasonal element and such irregularities as are present in the data.

If no allowance were to be made for year to year variations in the date of each week, the ratios would be tabulated by weeks, an average of the middle items computed, and the total adjusted to 5,200. Since it is necessary in the example given, however, to allow for the changing date of each week, the ratios are tabulated according to the day on which the week ends, as is illustrated in Table II, column 1. When two ratios appear for any day, they are averaged in column 2; if no ratio had been available for a certain day, a figure would have been interpolated from the preceding and following days. The fluctuations of these daily ratios are smoothed in column 3 by calculating the median of a moving five day period, placing the median opposite the third day of each period. In column 4, any remaining irregularities are eliminated by inspection, and the total is adjusted to 36,500. The seasonal indexes for any year are obtained from column 4; for example, the index for the first week of 1932, which ended on January 2, is 104.8, and for the second week is 102.5.

AN OPEN LETTER

Editor, Journal of the American Statistical Association:

Sir: The first sentence of the review of Woods and Russell's *Introduction to Medical Statistics* by a Mr. Kopf runs:

> In the preface Dr. Major Greenwood declares that the young medical men and women coming under instruction in the Division of Epidemiology and Vital Statistics of the London School of Hygiene and Tropical Medicine are taught to carry out with facility and confidence the statistical operations which a Medical Officer of Health must supervise.

Why Mr. Kopf should put so silly a brag into my mouth I do not know. What I actually said was that

> We hope, however, that we may give sufficient instruction to enable every student to understand and carry out the statistical operations which a Medical Officer of Health *must* supervise and to encourage many students to extend their knowledge. We believe that when a student has acquired facility and confidence in carrying out the simpler operations of statistical analysis, he will usually wish to go further.

No experienced teacher is fool enough—to put it no higher— to promise 100 per cent of successes. How on earth, for example, could I, or any other teacher, in a much longer course than we give at the London School of Hygiene expect to teach Mr. Kopf to make accurate statements? I see that, according to him, "on page 35 the definition of infant mortality is clearly wrong." I turn to page 35 and read "Infant mortality is generally measured as the deaths under 1 year per 1,000 births in the same year." This is not a definition of anything but an accurate statement of the practice of the General Register Office of England and Wales which it was Woods and Russell's business to explain to the students, just as it was their business to point out (on the following page) why the method of measurement was faulty.

Whether this book is or is not of educational value, is clearly a matter on which two opinions are possible. I do not think two opinions are possible as to the impropriety of the statements to which I have called attention.

I am,

Faithfully yours,

M. GREENWOOD

UNEMPLOYMENT—ITS ORIGIN AND ITS ELIMINATION

A dinner meeting of the American Statistical Association was held on Tuesday evening, March 29, 1932, at the Peg Woffington Grill Room, 413 West 23rd Street, New York City. Seventy-seven persons were in attendance. Dr. John B. Andrews, Secretary of the American Association for Labor Legislation acted as Presiding Officer. The general topic for discussion was "Unemployment—Its Origin and Its Elimination."

The first speaker of the evening was Dr. Arthur D. Gayer of Columbia University and the National Bureau of Economic Research. He spoke on "Public

Works as a Stabilizer of Employment." He pointed out that the proposal to use public works as a remedial measure is not new, having been advocated as long ago as 1909 in the British Poor Law report. The principle was also endorsed in 1923 by the President's Conference on Unemployment.

Because of the widespread discussion of the subject, it is often erroneously assumed that the volume of public works has been increased greatly during the present depression. As a matter of fact, this volume in 1930 was only a trifle greater than in 1929 and was below the level for 1927 and 1928. There would indeed have been a decline in 1930 had that year not been devoted largely to finishing up undertakings begun in 1929 when prosperity was still with us. True, Federal public works were more extensive in 1930 than previously, but state and local undertakings were curtailed. Further large declines in the latter lines also occurred in 1931.

If public works are to be used to stabilize the cycle, their volume, evidently, should be made to oscillate in a direction opposite to the ups and downs of private industry. In too many cases, what actually happens is that borrowing for public works in good times increases the volume of credit inflation and accentuates the boom.

One must not assume that all public works are susceptible of advancement or retardation. However, perhaps 50 per cent of the volume could, with advantage, be shifted from times of prosperity to times of depression. Although this volume would seem to be very small as compared to the deficiency in industry in a depression like the existing one, it is believed that the indirect effects of expanding employment might be sufficient to build up a volume of activity large enough to be of real moment. The question which has not yet been finally settled is the exact time at which public works ought to be centered in order to secure the best results. Some believe they should be speeded up early in the decline, others that concentration should be late in the depression. Little of a remedial nature can be accomplished unless plans are completed long in advance, for, normally, a considerable period elapses between the initiation of a project and the actual spending of the money for public works. The Federal Stabilization Board expects to make plans for six years in advance. It is hoped that states and municipalities will establish similar boards to perform like functions.

The recently observed curtailment in public construction has been primarily due to the fiscal difficulties under which the various branches of the Government have been laboring. Many hold that, under such circumstances, a public building program is not feasible. Actually, however, the probabilities are that, if the Federal Government were now to borrow five billion dollars for public construction, business would be greatly stimulated and existing taxes would increase to such an extent that the additional revenue would more than cover the interest on the new bonds issued. Many fear that expansion of public works would cause great waste of public funds. If properly managed, such enterprises would be conducted on a basis of expense lower than would be possible in boom times. At any rate, the Nation would get some return from the work of men engaged in construction. Now, we are compelled to support the unemployed by charitable measures, and the public gets nothing from them in return.

The second speaker of the evening was Beulah Amidon, Associate Editor of *The Survey* and *The Survey Graphic*. She dealt with "The Employer's Part in Stabilization." She began by pointing out that, before much headway can be made in stabilization through the action of the employer, the employer must be made to take an interest in the matter. A great deal may be accomplished when employers in general put as much energy into stabilizing production and hence employment as they now devote to problems of production, selling, and finance.

Numerous methods of cutting off the peaks and filling in the troughs have been found feasible. Some employers have found new uses for old products or have stimulated the use of these products at seasons in which there was formerly no demand. Other employers have found it profitable to produce special articles for delivery in seasons when their main line of business is dull. In some concerns production is budgeted so that it goes on regularly even though the demand for the products fluctuates. This plan works well when the product is non-perishable and easily stored. Attempts to stabilize production by increasing sales pressure in dull times, while helpful in the seasonal slumps of "normal" conditions, are of little use in an extended depression. A stabilization program is often difficult to work out and, in some respects, may prove costly. On the other hand, by reducing labor turnover, it represents an economy for the employer. Furthermore, unit cost of production is cut through assuring the worker of a steady job. He is not under the temptation to "stretch out" his work by loafing, breakage and so on; and he is willing to work at a lower wage rate when he is sure of a steady and hence higher annual income.

Many employers who have not been able to keep production steady have tried to "spread work," giving the whole force part time, instead of keeping a reduced force full time. When carried too far, this procedure may reduce wages so far as to break down living standards.

Experience indicates that employers can do much to lessen the hardships of technological change. They may reduce such unemployment by making the changes in planned and orderly fashion. They may assist employees to find new work or train them for new positions in the plant. Some concerns give a dismissal wage to employees laid off through no fault of their own. Employers cannot, however, eliminate all difficulties arising from technical change.

In dealing with unemployment, the far-sighted employer encourages two lines of attack under public control. The first is adequate public employment service to help workers find new positions promptly. A further step is to provide sizable reserves out of which some compensation may be paid an unemployed worker, just as accident compensation is paid an injured worker.

Wisconsin has recently passed a law making the accumulation of such reserves compulsory. The New York State Legislative Committee has endorsed the idea in principle but, as yet, no action has been taken. All the measures mentioned, while useful, are palliatives. They treat symptoms, not underlying causes.

Obviously, something more far-reaching is needed. A picture of our present chaos shows, on one side, producers with vast supplies which they are unable to sell; on the other, consumers in need of these supplies and unable to purchase

them. Some central agency to coördinate supply and demand is clearly necessary.

The third speaker was Willford I. King, Professor of Economics at New York University, who discussed the possibility of remedying unemployment through coöperation of employers and employees. He began by pointing out the fallacious nature of the theory that, under our present system, employers bear the risk to industry caused by the business cycle while employees receive steady wages and hence take little risk. As a matter of fact, whenever depression comes, either wage earners are laid off in great numbers or put on short time. Wage rates per piece or per hour are, indeed, maintained, but this is far from saying that weekly earnings are not cut sharply.

In the speaker's opinion, the common belief that it is desirable to maintain wage rates on an even keel during prosperity and depression is extremely unfortunate, for this practice of wage maintenance makes unemployment inevitable. Labor is subject to the laws of supply and demand just as fully as are other commodities. When the demand for other commodities shrinks, and, at the same time, prospective sellers maintain their holding prices unchanged, evidently the volume of sales diminishes. This situation explains fully the origin of unemployment.

When depression appears, the curve representing the demand for labor shifts downward and to the left, for employers are no longer able to sell their products at the prices previously prevailing. Since wage earners in general demand the same rates of pay per hour or per piece as formerly, it evidently follows that the volume of sales of labor will decline; in other words, part of the labor force will be unemployed.

There are only two possible ways of remedying unemployment: first, by strengthening the demand for labor; second, by reducing the subjective prices which laborers place upon their own labor. In a cycle in which there is no downward trend in the general price level, unemployment finally disappears because the demand curve for labor shifts upward and to the right. In the present depression, the general price level seems to have a sharp downward trend. It appears, therefore, that the only possible method of assuring a market for the total labor supply is for laborers to reduce their subjective wage rates. All devices, such as the dole, unemployment insurance, and the like, which tend to interfere with these principles of adjustment, prevent the elimination of unemployment and the recovery of business.

The speaker suggested that, if employers and employees both eliminated from their minds the idea that wage maintenance is desirable, it would be very easy to do away with unemployment, especially in those industries turning out products the demand for which is elastic. In such industries, the employers might well agree with their employees that the prices of products would be cut to whatever extent might prove necessary in order to keep the plant operating full time. To make this program possible, it would obviously be essential to slash both wages and salaries—perhaps very heavily. Acquiescence of the employee in a program of this sort should be secured by promising that dividends would be reduced in the same proportion as wages and salaries and that, whenever dividends were

again increased, wages and salaries would also be increased by a like percentage.

Instead of reducing the hours of labor per week, the employer should lengthen hours so that the weekly earnings of the employees would be sufficient to maintain them in comfort. The satisfactory operation of such a plan would be made much more probable if, during prosperous times, heavy reserves were laid aside to be used to subsidize both wages and dividends when depression appeared.

The last regular speaker on the program was George Soule, Editor of the *New Republic*, who spoke on "The Elimination of the Cycle as a Means of Lessening Unemployment."

Mr. Soule began by raising the query as to whether the plan proposed by the previous speaker for eliminating unemployment would work unless rents and interest charges, as well as dividends and wages, were reduced during depression periods. Mr. Soule believed that, as a matter of fact, it is not feasible to eliminate unemployment unless it is possible to eliminate the cycle itself, a result which does not seem possible unless we can install a system of planning and control.

The fundamental cause of the present depression appears to be that, during the last few years, there has developed a lack of balance between the volume of industrial equipment produced and the demand for the products of industry. Income has not been distributed in the proper degree. Industrial production and profits have advanced rapidly while wages and farm incomes have increased but slowly. The result has been a concentration of a large proportion of the income of the Nation in the hands of a small proportion of the population. The recipients of large incomes have been unable to devise methods of consuming their incomes in full and hence they have been almost compelled to invest in securities. This invested money has resulted in great expansion of productive plants and also in the volume of credit, but the demand for the products of industry has failed to keep pace with plant expansion.

If we are to avoid similar catastrophes in the future, it is imperative that we have a chance to plan and control the distribution of income and credit. Strong unions in all industries should be encouraged to push up all wages as high as the marginal concerns could afford to pay, and the higher cost concerns should gradually be eliminated. Farm incomes should be increased. Productive equipment should not be allowed to outrun the demands of the consumer. The supply of credit should be adjusted to the needs of business. In certain industries, such as house construction, the prices of the products should be lowered. The same would be true in the case of the output of public utilities. Utilization of land should be studied carefully and sub-marginal farm land should be eliminated from production. The tax programs and the budgets of various governmental organizations should be scientifically controlled. Foreign investments should be supervised. Concentration of income in the hands of the rich should be lessened.

The Swope plan for control of industry is wrong in that it proposes to put control into the hands of trade associations. These associations would proceed to reduce output—exactly the reverse of what is necessary. Control of industry should be in the hands of a Board of Experts representing the Nation as a whole and not merely an interested group. Industry should be consolidated in large

units. In some cases these units should be publicly owned and in some cases public supervision might be better. In many cases products could be standardized and the price-spread between producer and consumer thereby lessened. Nation-wide labor exchanges should be established. Unemployment insurance should be provided to prevent suffering among men who are thrown out of work.

The program just outlined is the minimum basis upon which it is possible to eliminate unemployment.

Several persons, speaking from the floor, raised various questions and made comments along a number of lines. One speaker pointed out that unemployment is an international and not merely a national problem and that it is a mistake, therefore, to consider international relations as unimportant.

Barnabas Bryan stated that the fact that unemployment is a resultant of the wage rate and the demand for labor is readily shown by a mathematical computation. If the United States Bureau of Labor Statistics' *Index of the Prices of Commodities at Wholesale* is divided by the average wage of all railway employees —a wage comprehensive enough to show the general level of wages in the United States—and if the quotients derived by this process of division are plotted, it will be found that the curve follows very closely that representing the volume of employment in the United States. Clearly, then, if one wishes to eliminate unemployment, one must vary the wage rate to correspond with changes in the commodity price level.

Leifur Magnusson mentioned the fact that the diversity in the movements of wholesale and retail prices during recent years has been less than has been the case in the past.

Several speakers brought out the fact that, if industry were to build up huge reserve funds to cover periods of depression, these reserve funds would have to be held in the form of cash or marketable securities. If kept in the latter form, the selling of these securities would presumably cause a great decline in their prices and would, therefore, result in a heavy loss to the fund. In answer to this contention it was pointed out that, if the reserve fund were held in high-grade government securities, such as Federal Reserve Bonds, money might be secured by borrowing on these bonds rather than by throwing them on the market.

The latter part of the discussion ranged around the fact that the charges required to maintain unemployment reserves would necessarily constitute a burden upon wages and that the wage earner should not be penalized when he moved from employment in one concern to employment in another concern.

WILLFORD I. KING

FORECASTING METHODS SUCCESSFULLY USED SINCE 1928

A dinner meeting of the American Statistical Association was held on Tuesday evening, April 26, 1932, in the Hotel Governor Clinton, 31st Street and Seventh Avenue, New York City. Two hundred and thirty-three persons were present. Dr. Edmund E. Day, Director for the Social Sciences for the Rockefeller Foundation, presided. The general topic for discussion was "Forecasting Methods Successfully Used Since 1928."

The first speaker of the evening was Dr. Lionel D. Edie, of the American Capital Corporation. He began by pointing out that success in forecasting either the bull market of 1925–1929 or the bear market of 1929–1932 does not demonstrate that the method used by the forecaster possessed any particular merit. In all probability, success was due merely to good fortune.

Before 1929, forecasters relied mainly upon mechanistic systems. The essence of such a system is that it can be communicated to another person who can then proceed to use the system successfully. Such systems are supposed to eliminate the judgment factor. Dr. Edie expressed the view that such methods lead only to futility, for judgment is the really essential feature of every successful forecasting system which is based upon anything other than pure luck.

The speaker illustrated his thesis by citing his own experience in regard to his prediction early in 1930 that commodity prices were destined to fall drastically. He made this forecast because, by talking with the leading central bankers of Europe and America, he had discovered that their mental attitude was such as to lead to deflation. It would be extremely difficult to arrange any mechanical system which would take account of these mental attitudes. Nevertheless, without understanding the philosophies of these bankers, one could have told nothing of what was going to happen.

Dr. Edie stated that a year ago he had predicted that deflation would continue until leaders of the inflationist school gained political control and forced the central banks to increase the supply of money and credit. The deflationists have engineered the depression to date. Now, in the United States, the "wild men of the west" are springing into the saddle and they will strive to direct the course of prices in the future. It is impossible to guess how effectively they can exercise their power.

A few years ago, statisticians everywhere were much interested in constructing very broad composite index numbers, believing that the breadth of these numbers would make them helpful in forecasting. The present tendency is to substitute the atomistic for the inclusive method—for example, if we wish to forecast the volume of steel production, we now attempt to measure each separate item in the demand for steel.

Another method of forecasting which has proven wholly futile is that basing its conclusions upon the composite opinion of the "best" minds. All that this method accomplishes is to warp the judgment of the statistician making the forecast, for the "best" minds usually are as badly informed about the outlook as the man on the street. To succeed, the statistician's judgment should be influenced as little as possible by popular emotion.

The second speaker of the evening was Paul Clay, Investment Counselor. He began by pointing out that white-haired forecasters are rare. He ascribed this situation largely to the fact that, every few years, industry passes from one era to another. In these different eras, different relations between industries and prices tend to prevail. Between 1863 and the present time, we have passed through no less than six such eras.

Mr. Clay stated that, after years of research concerning the cyclical movements of prices, he had built up five rules which he believed to be dependable. Between

1928 and 1931, however, all five of these broke down. The present indications are that this breakdown was caused by the fact that the rules were applicable to some eras but not to others.

Mr. Clay stated that he now felt that, in the past, he had underestimated the importance of the New York Stock Market itself in the industrial and financial affairs of the United States, and even of the world. This market, in fact, constitutes a secondary central bank for the United States, for it is the center at which all values can be liquidated on demand. The movements of the stock market represent the net result of the industry of the United States and a considerable proportion of the rest of the civilized world. Because of this conclusion, Mr. Clay has been led to construct a new index similar, in general, to the Dow theory, but not based upon the Dow methods. This index number he calls a psycho-technical index. It contains five principal elements:

1. A volume index number made by giving the sign of the price movement to the daily volumes, and accumulating the plus and minus movements.
2. Price movements of the twenty stocks having the largest volume of sales on a given day. This item is designed to cover pool activities.
3. A time index number made by adding 1 to the index for each upward day, and subtracting 1 for each minus day.
4. Resistance ratios designed to show the difference between liquidation and short sales.
5. The velocity of movements of stock prices.

The psycho-technical index built out of these five elements looks much like a price chart with the false movements eliminated.

It has the very distinct merit of often moving contrary to the course of the market itself. This index is not used independently, but rather in conjunction with the economic indexes which formerly constituted the chief reliance of Mr. Clay. In most cases, the indications of this mechanical barometer have proven more valuable than any conclusions arrived at on the basis of personal judgment.

The third speaker of the evening was Dr. Lewis H. Haney, of the Bureau of Business Research of New York University. He took the position that business forecasts have hitherto depended entirely too much upon statistical technique and have given too little consideration to the laws of economics. He felt that, though different eras might exist, economic laws ruled equally well in all eras. What the forecaster really needs is an abundance of statistical information and thorough training in economic principles. Unfortunately, in recent years, because of the vogue of "institutional economics" and "business economics," it has been more and more difficult to secure such training. Institutionalism is negative rather than positive in its attitude. It stresses description and exceptions to economic laws rather than the laws themselves. It tends to be "normative." Under the circumstances, the student is not sufficiently trained in positive economic principles. The weakness of business economics is that it gives too little weight to principles and too much weight to statistical facts. It has overemphasized the use of logarithmic charts and computed trends. These devices often mislead their makers.

To forecast successfully, one must analyze the facts in detail. It is, for example, important to keep track of the ratios of loans to deposits and of reserves to deposits. One must consider stocks of goods, production costs, trade conditions —both domestic and foreign—and everything else pertaining to the problem. It is important to observe relationships between supplies of raw material on hand and market activity. Margins between costs and selling prices are also extremely important.

Dr. Haney pointed out that, as long as business attempted to maintain wage rates at an abnormally high level, there was maladjustment, which was bearish—not bullish. He expressed the view that credit results from business and is not a cause of business. One cannot make more business by putting out more money. The maladjustments of 1929 have not yet been completed. Until this occurs, we cannot look for any sustained recovery.

He held further that empirical methods which depend upon precedent—upon the number of times that a certain relationship has recurred in the past—are inherently unsound. Scientific methods are the opposite of empirical methods. When these are used, there is no need of repeated demonstrations of their validity. One single test of a theory is ample. Economic principles can always be depended upon to hold true.

The last regular speaker on the program was Mr. James F. Hughes, of Charles D. Barney and Company. He explained that he was formerly a student of the relationships between economic activity and the stock market. He still believes that most of the relationships valid before 1928 will be found to hold true in the future. The big boom of 1929 must be considered an abnormality. Speculative enthusiasm caused the public in general to ignore the relationships dictated by common sense.

Even though the relationships between economic causes and market movements are fairly well established, the understanding of these relationships may be of little value to the speculator, for the time intervening between cause and effect is not sufficiently uniform to enable one to predict the turning point with precision, and, without this precision, it is not possible for the speculator to use the forecaster to make any considerable profits. Mr. Hughes stated, therefore, that he had been forced to depend more and more upon the action of the market itself as a guide to the location of turning points. In judging the action of the market, he depends not upon scientific reasoning but merely upon empirical rules based upon precedent. These empirical rules have proven very useful in assisting speculators to gain profits.

Mr. Hughes enumerated a number of the rules which he has found usually to hold good. He has observed that, after the course of the market has changed direction and covered a distance equal to 30 per cent or more of the movement just preceding, one must be on the lookout for a reversal in direction. Thus, after a selling climax in an active market, one should buy at once and sell as soon as the market has rallied by 40 per cent of the last break. When the market has risen for some time, and one of the speculative favorites collapses without any apparent reason, it is wise to sell out at once, for the rest of the market will probably follow.

Recently the market has been largely under the domination of seasonal forces, thus tending, for example, to decline in the second quarter of the year and to rise in the third. We may, therefore, look for a rally during the summer of this year. If business picks up, the rally may develop into a bull market, otherwise there will be another collapse. As a rule, the market rises between Christmas and New Year's. The market has habits because it is dominated by people and people have habits.

The remarks of the speakers were discussed briefly by Victor von Szeliski, of the Lehman Corporation. He pointed out that Dr. Edie was consistent in having no method to offer, inasmuch as Dr. Edie rejected the possibility of objective "indicators," and could recommend nothing better than using good judgment. This is a personal gift, uncommunicable by formula.

As one of the reasons why mechanical indicators so often failed, Mr. von Szeliski suggested that the interval, or time lag, between economic cause and effect was as a rule too small to permit the statistical detection of the cause before the effect followed. With reference to two of Dr. Haney's well-known indicators, the P/V line and the composite steel demand curve, he contended (1) That the former assumed a too simple relationship between P and V, viz., the regression coefficient of theoretical P on actual $V = 1$; thus, in periods of comparative price stability like 1921–1929, the P/V curve is in the main simply proportional to the reciprocal of V alone, and so "leads" the V curve by a half cycle of 1 to 2 years—a relationship observable between any cyclic curve and its inversion. (2) That the latter showed production of steel to have outrun demand for over a year: that this would have resulted in a tremendous inventory of finished steel products, which is not the case. Because steel production is almost wholly governed by specification, it is impossible for any such maladjustment as that found by Dr. Haney to occur. In point of fact, a carefully worked out demand composite does not show maladjustment. (There is maladjustment and inventory accumulation in the metal industry—at the mine, on the docks, in the hands of wholesale and retail dealers—but not where Dr. Haney finds it.)

Mr. von Szeliski denied any conflict between the rational economic approach of Drs. Edie and Haney, and the technical market methods of Hughes and Clay. The former tells *what*, the latter *when*. At least as far as stock speculation is concerned, the translation of our thought into *acts*, into buy and sell orders, must be governed by technical considerations. Economics may load the gun, select the mark, but technics pulls the trigger.

WILLFORD I. KING, *Secretary*

STATISTICAL METHODS IN ADVERTISING RESEARCH

A dinner meeting of the American Statistical Association was held on Tuesday evening, May 24th, at the Hotel Governor Clinton, 31st Street and Seventh Avenue, New York City. Fifty-five persons were in attendance. The chief topic for discussion was, "Statistical Methods in Advertising Research."

Malcolm Muir, President of the McGraw-Hill Publishing Company, acted as

Chairman. He stated that, in the existing stage of the depression, it has become necessary to scrutinize with unusual care all advertising appropriations. There is, however, a tendency to place advertising upon a basis more scientific than formerly was thought necessary.

The program was, in many respects, similar to that presented in Washington during the last Annual Meeting. It was felt, however, that since many New York members were not present at the Washington meeting, it was worth while to repeat, in essence, some of the papers previously given at the Annual Meeting. Students of the subject will do well to refer to pages 190 to 205 of the *Proceedings of the Ninety-Third Annual Meeting* (March, 1932, Supplement) for a summary of the material presented by Dr. Starch and Dr. Weld who were the first two speakers of the evening.

The third speaker of the evening was Dr. William J. Reilly, Marketing Consultant, who spoke on "The Law of Retail Gravitation." He began by stating that everyone realizes that the coming of the automobile and good roads has greatly extended the distance from which leading centers can draw trade. We have, however, heretofore lacked any mathematical measure of a city's drawing power.

Dr. Reilly presented a formula worked out by a study of actual data. He showed that a *city's drawing power for trade increases approximately in proportion to its population and varies inversely with the square of the distance from the city.* By aid of this mathematical law, it is possible to draw on a map the approximate borders between the districts contributory to given trade centers. The fact must be recognized, however, that dividing lines are not the same in the cases of all commodities. For example, the trading zone for higher priced articles is much broader than the trading zone for lower priced articles. This law of trade gravitation is also modified in many instances by the character of the roads, and by the existence of toll bridges and other obstacles.

In spite of these modifications and limitations, the law is, nevertheless, of great practical import. One important use is that it aids newspapers to know how far it is worth while to strive to extend circulation in order to aid their advertisers. Retailers can also see how far from their stores it is advantageous to spread advertising.

Mr. Oliver Everett, of the McCall Company, took the place of Mr. Lloyd Sweeting of the same concern in presenting a paper on the "Technique of Forecasting News Stand Sales of Magazines." He described the methods used by publishers in keeping track of the sales of various news stands. This is necessary in order that the publishers may send to each news stand the right number of magazines. If too few are sent, customers will not be supplied and circulation will shrink, while if too many are forwarded, the custom of making refunds to dealers will cause a heavy loss to the publishers.

The discussion was opened by Mr. W. B. Ricketts, Director of Research of Cowan & Dengler, Inc. He pointed out that the advertising statistician assumes that, under like conditions, the public will, at all times, respond in the same way to any given stimulus. In most cases, this assumption has proved valid.

He emphasized the fact that in the every-day research work of advertising agencies it is usually necessary, on account of cost and time factors, to obtain information that will indicate existing conditions with approximate correctness rather than data that more nearly approaches absolute accuracy. In many cases approximations that are correct within 5 or even 10 per cent meet the requirements satisfactorily.

In Mr. Ricketts' view, care in the framing of a questionnaire is extremely important, for the change of a word or two will modify, to a surprising extent, the results of the inquiry. In regard to sampling, he expressed the opinion that it is much more important to secure accurate items and a representative sample than to secure a large sample.

He concluded by stating that there was a great deal of interest at present in attempts that are being made to determine by test methods the relative effectiveness of different advertisements. A great deal of work has been done with the object of perfecting the technique of these testing methods and there are indications that real progress is being made.

The discussion was closed by Mr. Paul T. Cherington, Distribution Consultant. He enumerated seven different lines of advertising activity in which marked progress in statistical practice was being made. For example, he called attention to the fact that, owing to the work of the Audit Bureau of Circulation which has standardized and verified the records of the various periodicals, we now have much better statistics of newspaper and magazine circulation than were formerly available. Advertising men are now interesting themselves, not only in the quantity of the circulation, but also in its character.

The real problem before the advertising world today is to measure the value of the advertising itself, as distinct from the media employed. The chief obstacle in accomplishing this end is the difficulty of establishing good controls. Advertising sometimes produces unexpected results, for example, the advertising of an article by one store may cause the sales of the article to run up, not in the store that has advertised, but in nearby cut-rate stores.

The meeting adjourned.

WILLFORD I. KING

MISCELLANEOUS NOTES

Uptown New York Luncheon Meetings.—To date of this writing, there have been three meetings of the Uptown Luncheon Group of the American Statistical Association in New York City; all of these meetings have been held at the Fraternity Clubs Building. The object of these monthly meetings is to provide an opportunity for the New York members to hear an outstanding authority on some statistical or economic subject; discussion and comment form a very important part of the program after the speaker of the day has given his short remarks on the stated subject.

The first meeting of the Uptown Group was held on May 18. The speaker was Dr. Max Winkler, President of the American Council of Foreign Bond Holders, who spoke on the subject, "What Will Happen to War Debts and Reparations after July 1st?" Dr. Winkler outlined the circumstances which might be expected around the

middle of the year, and said that the only real hope of this situation was the Lausanne Conference. If reparations were to be paid, Germany would undoubtedly be bankrupt and her creditors would not only lose the reparations payments but probably would also lose the private debts owed by her. Cancellation or extreme scaling-down of reparations and war debts was advocated in preference to default or bankruptcy.

The second luncheon meeting was held on June 15. Mr. Howard Scott, former technologist of Muscle Shoals, spoke on "The Imminence of Social Change." Mr. Scott pointed out that the price system and scientific production could not exist side by side, and predicted "technocracy" as the social system of the future. He stated that, in the near future, one-half of the population of this country could support the entire population due to technological advances; the problem is how to readjust this disproportionate allocation of income for the best social good. Among other things, abolition of the competitive system and the price system were suggested; the "income" of the population would be apportioned on the basis of the mechanical energy used.

So great was the interest shown in this meeting, that the Planning Committee decided to hold a second meeting on this subject and obtain a somewhat different point of view. Mr. David Cushman Coyle, a well-known engineer in New York City, therefore spoke before the third luncheon meeting on July 13. His subject was "The Irrepressible Conflict—Business vs. Finance." Mr. Coyle essentially agreed with Mr. Scott on his factual data and the serious problems which are presented due to technological and social advances. Instead of abolishing the price system, however, he would change our monetary habits from that of a debt economy to one of a spending economy. On the premise that one-half of the population can guide and do the productive work necessary for the entire population, Mr. Coyle pointed out that we would have to support the non-working half of the population by a taxation-dole system, or by a greatly reduced working day so that at least part of this unemployed half of the population could have a job, or by spending our money on non-productive enterprises.

There was no meeting in August because of vacations. It is planned to resume these luncheon meetings on September 14. They will be held each month on the Wednesday nearest the middle of the month, for the next year, under the direction of Mr. Stanley B. Hunt, Assistant Secretary of the Association, Tubize Chatillon Corporation, 2 Park Avenue, New York City.

Progress Report of the Committee on Governmental Labor Statistics.—Since the completion of its study on public employment office statistics the Committee has been giving some attention to promotion of the final recommendations of the report. One of the more important suggestions is that local employment offices should not compile statistical reports of their work but that the entire function of compilation and publication should be centralized in state headquarters or at Washington. The work of each local office in this field would be restricted to the maintenance of a daily record of transactions, a copy of which would be forwarded at the close of the day's business to the central statistical authority. The central office, by large-scale machine methods, would secure a degree of refinement in the analysis and presentation of the data, quite impossible for the local units of the service. It is also claimed for this suggested procedure that it would ensure absolute comparability as between the work of local offices and state systems and would yield an accurate national report on the work of the public employment offices of the country, in contrast with the present national totals

compiled from the incomparable data of the various state services and federal offices.

In the Committee's own discussions and in its conferences with representatives of the International Association of Public Employment Services the objection was raised that the preparation of such a daily record of openings, applications, referrals and placements would impose too great a burden upon the local offices. The Employment Stabilization Research Institute of the University of Minnesota agreed to experiment with the procedure and for some time has been using in certain of the state employment offices the Committee's standard form for recording daily transactions. The Institute reports that, somewhat contrary to expectations, the record imposes little if any additional work upon the officials and that it seems entirely practicable. The Committee expects that the form will be tested also in the public employment office experiments at Rochester and Philadelphia and that arrangements may soon be made with a state statistical office to experiment with a centralized state compilation.

The Albany Chapter.—A dinner meeting was held on May 13, 1932, at 6.00 P. M. The gathering was addressed by Dr. Ralph G. Hurlin of the Russell Sage Foundation, who talked on "The Unemployment Survey in New Haven, Connecticut." A lively discussion followed. There were 24 persons attending the meeting.

Fifth Annual Meeting of the Boston Chapter.—The Fifth Annual Meeting of the Boston Chapter of the American Statistical Association was held in the auditorium of the Boston City Club on Thursday evening, May 12, 1932. Dinner was served at 7.00 P. M. There were 29 members and guests present at the dinner and business session, and 51 at the speaking session.

Business Session. The business session was called to order at 7.45 P. M. by the President, Mr. Leroy D. Peavey. The President spoke of the increasing interest in the meetings of the Chapter, and called attention to the advisability of holding joint meetings with other local organizations when this can be done to mutual advantage.

Officers elected for the ensuing year are:

President—Leroy D. Peavey, President Babson's Statistical Organization
Vice-President—Dr. John D. Black, Harvard University
Secretary-Treasurer—Roswell F. Phelps, Director of Statistics, Massachusetts Department of Labor and Industries
Counsellors—Edward A. Filene, William Filene's Sons Company; Professor J. F. Ebersole, Harvard Graduate School of Business Administration

The President appointed the following to serve as a Program Committee for the fall meeting:

Professor J. F. Ebersole, Chairman, Harvard Graduate School of Business Administration, Soldiers' Field, Boston, Massachusetts
Mr. K. B. Emerson, Business Research Department, United Shoe Machinery Corporation, 140 Federal Street, Boston, Massachusetts
Mr. E. L. Quirin, Babson's Statistical Organization, Wellesley Hills, Massachusetts

Speaking Session. The general topic for consideration at this meeting was "What Do Index Numbers Mean?" The speakers of the evening were Dr. A. G. Silverman, National Bureau of Economic Research, and Dr. Willford I. King, Secretary-Treasurer, American Statistical Association.

The following is an abstract of the address of Dr. Silverman:

What have all the measures grouped under the general heading of index numbers in common? Knowledge of the behavior of a group of elements is being sought. A cer-

tain economic realm, more or less clearly defined, is to be investigated, ordinarily to determine change over time. Further, a unit of commensuration is required. Finally, some form of combination of the elements in this economic realm, expressed in this common unit, is to be made. Here we have the more general problem of economic measurement. So-called index number problems occur even in the derivation of what commonly are assumed to be *individual* series or items—an assumption that may or may not be safely made.

By including many different conceptions and interests in economic measurement within a single term, "index numbers," purporting to embody a common technique, confused and low standards are perpetuated. The fact that an investigator may not be, and often cannot be, quite clear as to his intentions does not help matters. He may ask a specific question in the form of a verbalization of a numerical expression, but he does not thereby overcome his actual vagueness on the subject of what is to be measured, and why. Unfortunately, too, seemingly harmless mathematical expressions, especially if used in the measurement of movements over time, may sometimes imply more than is intended. And without some clarity regarding the economic realm to be measured, there is little chance of selecting the appropriate form of combination. In the present state of our knowledge of economic change and connection we often can do no more than search for common categories of change and sequence.

The unit of commensuration presents even more difficult problems. This is obvious for the combination of quantities, expressed in diverse physical units; it is less obvious for prices expressed in a common money unit. But prices, too, relate to diverse physical units and are likewise incommensurate. The difficulty is especially acute for stock price indexes.

When can sampling be logically employed in the construction of index numbers? When are we justified in calculating index numbers for relatively long periods of time? Are quantity, price, and other indexes similar in nature?

These questions overlap and to that extent their answers are inseparably tied together. In attempting to answer them it is useful to start with the conception of "common causation." This is a shorthand expression for signifying that a cause or set of causes operates uniformly on, or through, individual members of a class. One of the chief objectives in the construction of index numbers, at least when viewed historically, has been the measurement of the influence of "common causation." Measurement of monetary changes took the form of calculation of averages, on the consideration that, since individual price changes reflect the combined influence of particular factors peculiar to single or limited groups of prices and the common monetary factors, the average should reflect the net impress of the monetary factors alone.

In proving the existence and the extent of such "common causation" we may take the theoretical or the empirical route, or both, as in price measurement. It may be that a good deal of our work must look toward the development of technique designed to establish the existence and limits of the effects of common factors. We may not know what the economic realm delineated by "common causation" may be, except that it exists or is assumed to exist. If work must be with sample data, an average is justified only when there is evidence of the operation of common forces, if the average is to have the "reality" postulated. And then it is necessary to strike an average of a sufficiently large number of series to make possible the mutual cancellation of compensatory influences and thus to reveal the effects of the common factors operating on the system. The average in this case has some reality, or perhaps *a priori* validity, and is not merely a summarization characteristic.

The assumption of "common causation" may or may not be justified. Whenever it can be proved as invalid or is not substantiated, either on theoretical grounds or by a statistical examination of changes in the series to be combined, Dr. King should agree that the term index number is at least questionable on the basis of his definition of index numbers as samples. Then, it would follow that the term is improperly applied to practically all quantity indexes and to indexes calculated for relatively long periods; unless we are to select samples and strike averages on a frankly empirical basis.

Only the practically complete measurement of the economic realm, or the conception of "common causation," proceeding largely on *a priori* grounds, can give us a sound logical basis for the construction of index numbers. Thus I hold that sampling is relatively meaningless for quantity indexes over a broad economic realm, except perhaps for measuring short or long wave cycle movements. Similarity of movement in individual quantity series cannot be postulated with reference to secular movements, and we are far from possessing complete data. For very long periods of time sampling is inappropriate for price indexes. Though the monetary factors operate continuously, they do so on ever-changing members, which not only renders the members increasingly incommensurate, but precludes sampling. Even if we had an invariant unit referring to quantity of satisfaction, sampling would be logical only on the assumption of "common causation." For prices, this assumption might be, but for quantities it would not be, justified. Index number construction from samples has much greater validity for the measurement of cycle movements. Here, too, the difficulties of obtaining a unit of commensuration do not appear to weigh so heavily on the makers of index numbers. Often series are available for cyclical indexes, but not for long term measures, and the attempt to utilize the same series for both purposes may be self-defeating.

Group weights are justified only if it can be established that there is sufficient similarity of movement between the elements combined and the combination of the remaining part of the individual elements in the system. And here *a priori* or theoretical reasoning appears almost indispensable, for empirical testing is difficult and often impossible. Census categories used as a basis for applying group weights are totally uncritical, especially in the field of quantity indexes. The aggregation of individual quantity series and group indexes of such series often seems to proceed on the basis of what may be called the "accumulation of insufficiency," rather than on that of compensation of individual errors to reveal a common characteristic or to give a representation of a total.

Most makers of index numbers have in mind, although they may not explicitly state it, a distinction between representativeness and sampling. I believe that "representativeness" refers to a recognition that its determination, as compared with the kind of sampling that is based on a logical theory such as is discussed in this paper, is largely empirical. Often the assumed representative character of a series or group of series is a rationalization of what we don't know or are unwilling to admit. The appropriate comparison for determination of representativeness is between the items combined and the combination of those not to be included. To prove that the index numbers calculated move like another series of index numbers probably means that both have been constructed from much the same elements. Even if the part could be proved representative of the larger total, this would not necessarily mean that the part was a good index number, for both the part and the larger total may be bad for the same or similar reasons. The very idea of representativeness may be another way of pointing to the fact that the required homogeneity of composition, or common con-

figuration, of the total and the index numbers referring to changes in the total are nonexistent or confused, so far as our knowledge goes.

It is not here intended to insist that sampling and averages are always the central problems—or even always theoretically involved—in the combination of individual elements. In a sense, the use of group weights already contradicts the logic of sampling, and indicates that a total of contributing members is thought to be measured. There may be little or no similarity of movement among the different elements, and yet it may be desired to combine them. Combination may be intended as the empirical-statistical equivalent of a reduction in the number of equations of a formal mathematical system. Such reduction may be accomplished through summation (or aggregation) or by substitution and the use of equivalents. The technical procedure and criteria in *The Making of Index Numbers* are largely based on the logic of the equation of exchange, represented in Professor Fisher's *Purchasing Power of Money* as indispensable to the solution of the Walrasian system of equations defining an economic equilibrium. One may for certain purposes postulate the economic significance of given categories and assume their qualitative continuity, and then calculate, say, quantity indexes for these categories, to be representative of whatever economic connections can be established between the movements of the index numbers and of other elements in the economic system, no more nor less. For example, I have calculated index numbers of British export and import quantities from 1880 to 1913 for restricted classifications given by official sources in order to reduce the numbers of time series to be analyzed. Such index numbers mean much more for study of cyclical fluctuations than for secular trend. Also, the greater the degree of combination the less is the possibility of establishing meaningful relations.

Summation is not always undertaken to compensate errors, but may be for the purpose of covering a larger field. Measurement of changes in welfare for the total economy, or for given parts of it, may be the objective, with a view to evaluating consequences of economic evolution or perhaps in the interest of redefining social objectives. Social budgeting or accounting is made inevitable by limited resources and by the limited possibility of satisfaction of human wants. Quantity indexes generally relate to aggregates. Conceptually they are not averages, even though averages may be used in the technical procedure of obtaining them. What quantity indexes report is the changing number of composite standard units. Price indexes, too, may be viewed as rates with reference to a composite standard, a dollar's worth or a basket in a given year; and from this standpoint they may also be considered as aggregates.

The difficulties presented by a lack of a homogeneous unit of commensuration in economics, and the question whether price and quantity indexes are correlative in respect of permitting meaningful combination (for purposes of obtaining some value index) and in respect of formulae employed in their construction, were briefly referred to.

The second speaker of the evening was Dr. Willford I. King, Secretary of the American Statistical Association. He began by taking issue with the old idea that index numbers represent merely, in some vague sort of way, the general movements of the data in the totality. He pointed out that the same totality of data may be used to answer many diverse questions and that the exact answer to each question may be represented by a different set of relatives derived from the data.

The speaker emphasized the fact that series of ratios derived from sample data may be very different from those derived from complete data. Since these two concepts are different it is well to give them two different names, the term "index numbers" being reserved for the indicators derived from the sample data. These indicators, or

index numbers, can never be expected to answer with precision the question answered by the relatives representing the totality. However, with reasonably adequate sampling, the course of the index number may approach rather closely to the course of the relatives just mentioned. In such case, the index number is satisfactory.

He agreed with Dr. Silverman that the use of sampling is not legitimate unless most of the items in the totality are dominated by a common cause. He held, however, that such domination by a common cause is not restricted to price data—for example, the yields of grain on the different farms of the United States are affected by developments in methods of cultivation, by the weather prevailing, and by the variety of seed used. It is then perfectly feasible to secure reasonably accurate estimates of grain yield in the country as a whole by securing reports from well selected sample farms.

To be useful, the sample must, of course, be representative. It may be safely assumed that pot roast is representative of beef in general and will be influenced by the various forces affecting beef. One cannot be so sure, however, that sugar is representative of foods in general.

Quantity indexes are neither more nor less reliable than price indexes. To compute quantity indexes, it is necessary to use arbitrarily defined units as, for example, dollar's worths in a base year. Experience indicates, however, that, when the number of items entering into the index number is rather large, the results are not materially affected by the arbitrary nature of the unit, for, in most cases, a change in the definition of the unit does not affect markedly the course of the index number. The price index is subject to all the weaknesses inherent in the quantity index for, before prices can be averaged, one must be sure that the prices apply to definite quantities, hence the arbitrary characteristic of the quantity unit enters also into the price index.

In all statistical studies, it is essential that the question asked be made very specific and that all units used be defined with precision. This general rule applies, of course, to index numbers but is no more applicable to them than to other types of data.

While index numbers are not instruments of precision, they do, nevertheless, in hundreds of cases, yield results which are close enough to the truth to answer all practical purposes. The use of index numbers is therefore extremely helpful to the general public as well as to the scientist.

Following the addresses, there was spirited discussion from the floor.

ROSWELL F. PHELPS, *District Secretary*

The Chicago Chapter.—The final dinner meeting of the Chicago Chapter for the 1931–1932 season was held on Tuesday, April 19, with 58 in attendance. At this meeting the annual reports of committees were heard and the election of officers and directors for the season took place. Professor Henry Schultz of the University of Chicago was elected President, Mr. John L. Sweet of the Federal Reserve Bank, Vice-President, and Mrs. Bernice Lamb of the Federal Reserve Bank was reëlected Secretary-Treasurer. Directors for the coming year are Professor C. A. R. Wardwell of Northwestern University, Miss Marion Mead of the Illinois Chamber of Commerce, and Mr. H. L. Jones of the Illinois Bell Telephone Company.

The general topic for this last meeting was "Shall We Legislate a Higher Price Level?" The subject was ably discussed by Mr. Henry A. Wallace, Editor, *Wallace's Farmer*, and Mr. George A. Putnam, Economist, Swift and Company. Mr. Wallace is one of the leading agricultural economists of the country and has been actively engaged in obtaining Congressional action to stabilize prices at a higher level. His discussion followed these lines, while Mr. Putnam, who is a money and banking expert, focused his talk on the relationship of the price level to the gold standard.

Activities of the Cleveland Chapter.—The Business Statistics Section of the Cleveland Chapter had its final meeting for the season in April. Nine meetings were held during the season with an average attendance of 23 members and guests.

The annual banquet of the Cleveland Chapter was held on April 8 at the Chamber of Commerce Club. The attendance was in excess of 150. The meeting was held in coöperation with the Ohio Group of Statisticians.

We were particularly fortunate in securing as speakers, Professor Irving Fisher, Mr. John Scoville and Colonel Leonard P. Ayres.

Professor Fisher addressed us on "Some First Principles of Booms and Depressions."

Mr. John Scoville, Statistician for the Chrysler Corporation, chose as his subject, "The Behavior of the Automobile Industry in Depression."

Colonel Ayres, Vice-President of the Cleveland Trust Company, described some "New Source Material for Business Cycle Research."

Officers for the 1932–1933 year are as follows: President—Howard W. Green, Cleveland Health Council; Vice-President—D. A. Hill, The Ohio Public Service Corporation; Secretary—E. A. Stephen, The Ohio Bell Telephone Company.

The Connecticut Group.—A local chapter of the Association is being organized in Connecticut. Chester H. Whelden, Jr., of Yale has been appointed District Secretary. Meetings will start early in the fall and will probably be held in rotation in Bridgeport, New Haven, Hartford, and New London.

The Philadelphia Group.—The Philadelphia Statistics Group held its third and final meeting of the academic year on May 6. The topic, "The Present Depression and Steps Toward Better Planning of Production," was discussed, with Dr. Leo Wolman of Columbia University and the Amalgamated Clothing Workers of America speaking on "Some Aspects and Consequences of the Present Depression," and Dr. Joseph H. Willits, Director of the Industrial Research Department, University of Pennsylvania, speaking on "Steps Toward Better Planning of Production."

Meetings of the Pittsburgh Chapter.—At the Chapter Meeting on April 20, Dr. Francis Tyson read a very well-prepared paper on "Price and Cost Factors in Business Recovery." Mr. J. E. Webster of the Westinghouse Electric and Manufacturing Company gave a short discussion on "Mass Production as a Basis of Prosperity," while Professor McCabe of the University of Pittsburgh commented on Dr. Tyson's paper. Approximately thirty attended the meeting, which developed into an unusually stimulating discussion.

The May meeting, which was held on the 26th, was very interesting and attended by approximately thirty members. Dr. Watkins gave a fine presentation on "This Depression and the Next."

At the June 23rd meeting, discussion was led by Dr. Montfort Jones and Dr. George McCabe, of the University of Pittsburgh, on the subject of "The Probable Effect of the Federal Board's Policy in Purchasing Government Securities." Approximately thirty-five attended the meeting, with Mr. Nevin, local head of the Federal Reserve Bank, presiding.

On July 28 the Chapter reviewed Colonel Rorty's recent paper in the *Harvard Business Review* on "How May Business Revival Be Forced." Mr. Bervard Nichols of the Bureau of Business Research led the discussion. Mr. Nevin again presided.

Mr. T. H. Gerkin, Resident Editor of *Iron Age*, has been elected Assistant Secretary of the Chapter.

The San Francisco Chapter.—Four meetings were held by the San Francisco Chapter during the past fiscal year, with average attendance of 35 at each meeting.

The election of officers and general business meeting of the year was held on Friday, October 30, 1931, at the Bellevue Hotel. Dr. Henry F. Grady, Dean of the College of Commerce, University of California, and Trade Adviser to the San Francisco Chamber of Commerce, spoke on Dislocation of World Economics.

Dr. Grady developed the view that the present acute disturbance of world economic equilibrium is due primarily to the War, the revolutionary development of machine technique, and the disintegration of political organization. The War accentuated nationalism in a world economically international. Instruments of economic armament have been multiplied, and have taken the form of embargo tariffs and export bounties, and have caused congestion in commodity and gold flow with the resulting serious world trade dislocation and distress.

The second dinner meeting of the year took place December 11, 1931, with Dr. Norman J. Silberling of the Silberling Research Corporation as the principal speaker. The topic discussed was the Business Outlook for 1932.

Dr. Silberling reviewed some of the major causes of the present depression, but could offer little if any encouragement for improvement during 1932. He regarded the present situation as an aftermath of the War and stated that this might be regarded as the second installment of the costs of the War. The condition of various industries and the financial situation were examined at length.

The meeting of February 4, 1932, was a discussion of the Silver Problem from the Chinese Point of View by Theodore J. Kreps, Associate Professor of Business Administration at Stanford University.

Dr. Kreps had just returned from several months' travel in the silver-using countries and had made a first-hand study of the effects of falling silver prices. He summarized the effects of the decline in the price of silver on Chinese trade, general prices and wages. He pointed out the stimulation of Chinese industry which had resulted from the mild inflation, and spoke of the attitude of Chinese business men toward the problem.

The topic for the meeting of May 12, 1932, was A Critical Appraisal of Indexes of General Business Activity, discussed by Maurice I. Gershenson of the Silberling Research Corporation and Robert W. Bachelor of the Federal Reserve Bank of San Francisco.

The speakers stated that the continued drop in business activity during the past three years had shown a need for a critical examination of indexes which attempt to measure changes in business from month to month. Fundamental assumptions involved in indexes of business were examined from the standpoint of methods of construction such as computation of trend, isolation of seasonal movements, smoothing of data, combining series with varying magnitudes of fluctuation, weighting of each group entering into the composite, representativeness of the data used as a sample of all economic activities, and handling of habitual leads and lags of some series compared with others. Typical methods of constructing currently used and supposedly representative measures of business activity were then examined in view of the principles developed. These indexes were then classified as to their actual significance and reliability.

United States Bureau of Labor Statistics.—The Bureau's studies of wages and hours of labor in air transportation, the hosiery and underwear industry, the boot and shoe industry, the manufacture of woolen goods, and the dyeing and finishing of textiles have been completed and summaries of the data obtained are being prepared for publication. Reports on wages and hours in cotton-goods manufacturing, automobile repair shops, gasoline filling stations, the slaughtering and meat-packing industry, and metalliferous mining have been carried in recent issues of the *Labor Review*. Field work on the surveys of wages and hours in the lumber and tanning industries has been completed, and the gathering of data for the clothing and the motor-vehicle industries is under way.

The investigations of the effects upon employment of technological changes in the electric light and power and the automobile-tire industries have been finished and the information obtained is being prepared for publication. A similar investigation is now in progress for mail transportation.

A bulletin giving the text of Federal and state legislation relating to public and private employment offices is being prepared.

The annual review of industrial disputes in the United States, covering 1931, was published in the *Labor Review* for June, 1932, comparative figures also being given for each year back to 1916.

A study made by the Bureau of operations during 1931 under state old-age pension laws was published in the June, 1932, *Labor Review*.

Another article of special interest, published in the July *Review*, gave the provisions of Federal and state anti-injunction laws in labor disputes.

The series of articles on wages in foreign countries, based on reports furnished by representatives of the Department of State, is being continued in the *Labor Review*.

Publications of the Women's Bureau.—Preliminary reports on two studies have recently been published by the Women's Bureau of the United States Department of Labor. The first deals with the economic status of women engaged in the manufacturing of wearing apparel in Connecticut in the fall of 1931. The pay-roll data for hours and wages that formed the basis for the Bureau's report were obtained from 106 firms employing 10,009 workers—64 per cent of the state's wage-earners in the clothing industry, according to the 1930 Census of Occupations. With the exception of the hat factories, these were chiefly woman-employing plants, more than three-fourths of the wage-earners being women. Pay-roll records were taken for the week recommended by the firms as most nearly normal or as full-time as any.

In addition, a preliminary report on a state-wide survey of North Dakota has been published. The report covers 204 establishments located in 21 cities and towns and employing 1,742 women. The largest group of establishments included were hotels and restaurants, which made up more than two-fifths of the work places visited and employed nearly a third of the women. Stores, telephone exchanges, laundries, manufacturing establishments, and beauty shops were other industries covered by the survey. The preliminary report is confined to wages and hours of one pay-roll period of the women employed. The complete report will include year's earnings, working conditions, and facts as to age, marital condition, and length of service.

Recent Publication of the Bureau of Agricultural Economics.—*Factors Affecting the Price of Rice*, published under date of April, 1932, as U. S. Department of Agriculture Technical Bulletin No. 297, is another bulletin in the series of price analysis publications relating to individual farm commodities. Previous bulletins in the series have dealt with oats, hogs, and cotton.

The Canadian Census of Merchandising and Service Establishments, 1931.—The first reports for the Canadian Census of Merchandising and Service Establishments have been released and the preliminary tabulation is proceeding rapidly for the larger cities. The progress so far reported compares favorably with the results secured for the United States Census of Distribution which commenced a year earlier than the Canadian survey. The Canadian Census was based on names and addresses of establishments secured by regular census enumerators. Schedule information was secured almost entirely through the mails, although field workers were engaged to secure reports from delinquent establishments. It is expected that the Census, when completed, will furnish a comprehensive picture of merchandising in Canada. The system of classification in the Canadian Census has closely followed that of the United States Census of Distribution, by which procedure it is hoped that comparisons between the two countries will be facilitated.

Business Research Council.—The Business Research Council has recently published a report under the title *Economic and Business Research in American Colleges and Universities*, which presents the results of a census of research projects taken during the spring and summer of 1931. Donald R. Belcher, a member of the Business Research Council through appointment by the American Statistical Association, is Chairman of the special committee responsible for this publication.

The report covers research projects completed during the academic year 1929–1930 and projects in progress during the year 1930–1931. In all, 620 projects are listed, classified according to subject and cross-indexed for ease of reference. In addition to the title of each project, a brief description of the nature and scope of the research is given, together with information concerning status of the work, availability of results, and medium of publication where results have been published. Both investigations undertaken by organized research or other bureaus and those made by individuals associated with the institutions are included.

The report also lists some 80 recurring series of data compiled by the institutions, giving in each case the frequency of compilation, availability, and medium and frequency of publication, where data are published.

Ninety-two separate institutions reporting research in business and economics are included. In many instances reports were received from several schools or departments of a given institution, so that the number of essentially independent organizations for which data are given is 108. A list of these institutions, together with titles and chief fields of interest of organized research bureaus and details concerning their periodic publications, are shown.

Copies may be ordered by writing to the Business Research Council, 20 Vesey Street, New York City.

Activities of the National Distribution Council.—The National Distribution Council of Washington, D. C., an organization of research men interested in marketing problems, has just completed its second year of existence. A series of seven dinner meetings and six luncheon meetings was held during the past year. The attendance was exceptionally good and much interest was shown in the various papers presented. In each paper an attempt was made to present material which would contribute to the knowledge of the subject. The following program gives an idea of the scope of the work of this organization and the character of its membership.

During the first year all dinner meetings were held at the Cosmos Club and the luncheon meetings at the University Club. During the year just past, all meetings were held in the new building of the Brookings Institution.

DINNER MEETINGS—*1931–1932*

October—New Methods of Allocating Costs of Distribution, by Wroe Alderson, Department of Commerce.

November—The Relation of Stocks of Commodities to the Business Depression, by Dr. Fred W. Dewhurst, Department of Commerce.

January—Distribution and Consumption of Coal, by Dr. Harvey Young, Bureau of Mines.

February—Trade Statistics and Price Stabilization, by Dr. W. H. S. Stevens, Economist, Washington, D. C.

March—An Analysis of the Problems Connected with the Marketing of Radios and Radio Equipment, by Dr. N. H. Engle, Expert in Marketing, Bureau of the Census.

April—Agricultural Taxation and Some Relations to the Crop Surplus Problem, by Dr. Eric Englund, Department of Agriculture.

May—The Economic Meaning of Hand-to-Mouth Buying, by Dr. L. S. Lyon, Brookings Institution.

LUNCHEON MEETINGS

November—The Marketing Activities of the Bureau of Foreign and Domestic Commerce, by Dr. Frank Surface, Department of Commerce.

December—The Marketing and Other Activities of the Brookings Institution, by Dr. L. S. Lyon, Brookings Institution.

February—Marketing Activities of the Bureau of Agricultural Economics, by Dr. E. J. Working, Department of Agriculture.

March—An Analysis of Drug Store Transactions, by Mr. B. B. Aiken, Department of Commerce.

April—Marketing Activities of the Federal Trade Commission, by Mr. Martin Behrens.

May—Annual Business Meeting.

The Joint Meeting of "Section K" and the Econometrical Society.—The attention of our readers is called to the highly profitable and interesting meeting of Section K (economic and social sciences and statistics) of the American Association for the Advancement of Science at Syracuse, June 20–23, jointly with the Econometric Society. In attendance, quality of papers and discussion, and general interest, this was far ahead of the previous meetings of the two organizations. A complete report appeared in *Science* for July 22.

The Brookings Institution.—There have been the following changes in the personnel of the Brookings Institution:

Dr. Leverett S. Lyon has been made Executive Vice-President.

Mr. William F. Willoughby has retired as Director of the Institute for Government Research, and is succeeded by Dr. Arnold B. Hall, recently President of the University of Oregon. Mr. Gustavus A. Weber has retired, and Mr. Herbert Wilson has accepted a position with the United States Bureau of the Budget.

In the Institute of Economics, Dr. Robert R. Kuczynski, Mr. Max Sasuly, and Miss Adelaide R. Hasse have resigned.

The following appointments have been made to the staff of the Institute of Economics: Dr. Maurice Leven, formerly with the Committee on the Cost of Medical Care; Dr. Clark Warburton, formerly with the research staff of the Federal Reserve

Board; Dr. Horace B. Drury, formerly economist with the United States Shipping Board; Professor Chester B. Pond of Lebanon Valley College; Mr. V. S. Kolesnikoff, formerly with the National Bureau of Economic Research; and Mrs. Umber Arthun Warburton of Atlanta University.

The following persons have been awarded research Fellowships for the academic year 1932–33: Aaron V. Abramson, Milner Fellow, Brown University; Robert H. Connery, Graduate student, Columbia University; Howard A. Mackenzie, Teaching Fellow, University of California; Valentine S. Malitsky, Graduate student, University of Minnesota; Harold W. Metz, Graduate student, Yale University; Lionel V. Murphy, Instructor, University of Oklahoma; Stanley I. Posner, Research Assistant, University of Chicago; Irma M. Rittenhouse, Assistant Editor, Encyclopaedia of the Social Sciences; Boris B. Shishkin, Graduate student, Columbia University; Ernst W. Swanson, Research Fellow, University of Chicago; Cecil H. Tolbert, Cowles Fellow, Yale University.

Consulting Fellowships have been awarded to Thomas S. Barclay, Associate Professor, Stanford University, and to Abraham D. H. Kaplan, Professor, and Director of Social Studies, Bureau of Business and Social Research, University of Denver.

Personal Note

Mr. R. E. Watts, for many years Head of the Judicial Statistics Branch of the Dominion Bureau of Statistics, Canada, has recently been superannuated. Mr. Watts contributed an article on "The Influence of Population Density on Crime" to the March, 1931, number of this Journal.

MEMBERS ADDED SINCE JUNE, 1932

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Hale, Roger F., Bureau of Agricultural Economics, Washington, D. C.

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Neuberger, Roy R., Halle and Stieglitz, 25 Broad Street, New York, N. Y.
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Schavel, John, Jr., Brooklyn Borough Gas Company, Mermaid Avenue and West 17 Street, Coney Island, N. Y.
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Stevens, Robert E., Student, Cornell University, 101 South Baker Hall, Ithaca, N. Y.
Stover, Albert A., Jr., Brooklyn Trust Company, 177 Montague Street, Brooklyn, N. Y.
Sweet, John L., Federal Reserve Bank of Chicago, 164 West Jackson Boulevard, Chicago, Ill.
Taeuber, Dr. Conrad, Mount Holyoke College, South Hadley, Mass.
Thompson, Dr. Tracy E., Bureau of the Census, Washington, D. C.
Waters, James J., United States Bureau of Investigation, Washington, D. C.
Willits, Dr. Joseph H., Wharton School, University of Pennsylvania, Philadelphia, Pa.
Wilson, Alfred J. P., Research Engineer, 538 East 21 Street, Brooklyn, N. Y.
Wilson, Winfred P., Accounting Department, Statistician's Section, Michigan Bell Telephone Company, Detroit, Mich.

REVIEWS

Economic Stabilization in an Unbalanced World, by A. H. Hansen. New York: Harcourt, Brace and Company. 1932. ix, 384 pp.

Professor Hansen covers almost every conceivable subject in this lengthy and very useful volume. Russian dumping, reparations, the Bank for International Settlements, wage policies, unemployment insurance, and the optimum theory of population are a fair sample of the diversity of subjects treated; but the author is not always successful in knitting the story together. Although the volume represents a large amount of industry and much careful thought, any writer, courageous enough to embark on so ambitious a program without devoting a generation to its consummation, exposes himself to endless criticism. In what follows, several important questions are raised concerning Professor Hansen's treatment. Space does not permit a discussion of numerous points of agreement.

Reparations are held to be the decisive factor in the determination of Germany's economic future. It is contended that Germany is deprived of one dollar of capital for each dollar collected from the taxpayer for reparations, and that the consumption standards of Germany have not been reduced as a result of reparation payments. In view of Germany's tax system, which is characterized by large consumption taxes, it is especially difficult to understand why a *large* part of the taxes collected for the payment of reparations does not reduce consumption rather than savings. Moreover, no evidence other than reference to well-informed opinion is offered that consumption standards have not suffered. One may conceivably argue that the necessity of paying reparations explains the large influx of foreign capital, and, therefore, that the reduction in consumption standards suffered as a result of the payment of taxes for reparations, is to be offset by the gain in purchasing power to be associated with the influx of capital. It may also be contended that the payment of reparations induced conditions favorable to an increase in the export trade, and, therefore, that employment was increased, and the loss of goods involved in the payment of reparations is to be associated with an increase in output and employment rather than a reduction in consumption. But even if this position were taken, it might still be maintained that the standard of living was reduced as a result of the payment of reparations. In other words, those who are taxed for the purpose of paying reparations, even if employment and purchasing power are increased as a result of the payment of reparations, would consider that their standard of living had been impaired.

One more point on reparations. Hansen admits that reparation payments have tended to depress prices in Germany, but then he goes on to say that *until 1929–1930* the pressure tending to depress German prices was offset by the upward tendencies resulting from the inflow of capital. But the developments of the last two years are not easily explained in that manner. Germany's losses of gold and foreign assets have been much beyond the most optimistic anticipation of those who view the loss of gold by Germany and an influx into the United

States and other countries as the process by which reparation payments are induced. In spite of the fact that Germany has exported virtually all of its gold and sold most of its foreign assets and the German Government has taken drastic measures to reduce prices and wages, the decline in prices in Germany has not been more precipitate than in the United States.

Much has been said in recent years concerning the problem of capital equities, fixed costs, and the like, especially in relation to the problem of international competition. On this question, Hansen expresses himself in no uncertain terms and, in the writer's opinion, assumes a position that is clearly fallacious. He contends that cost accounting is not concerned with gifts and windfalls, and that the German industrialist cannot produce more cheaply because he has been freed of the necessity of paying interest on bonds as a result of the inflationary episode. But this is just exactly what German firms can do. In so far as they are relieved of fixed charges, they can sell abroad more cheaply than they otherwise would. Hansen contends that the reduced equity of bondholders is to be offset by an increased equity of stockholders. But stockholders do not have to be paid as a condition to staying in business; bondholders, on the other hand, are paid unless the directors are content to appeal to the bankruptcy courts. On the other hand, there is some truth in Hansen's position although he does not make this point (which follows); and English authorities, in particular, neglect it: A default on bond interest, whether it is the result of inflation or reorganization, raises costs of industry later, for the borrowers (in Germany, say) are forced to pay a higher price for capital later. Therefore, it may be held that the gain to be associated with payments on capital already obtained, is to be offset by losses to be associated with the increased cost of obtaining capital in the future.

Hansen discusses the problem of prices at many points. In his opinion, declining prices, even if associated with improved technological conditions and increased productivity per capita, have unfortunate results. A reduction of costs and prices may result in the expenditure of an increased amount of money for the products of industries profiting from a reduction of costs, and for that reason, he argues that other industries, unable to bring down costs, would be affected adversely, for a smaller amount of money would be expended for the products of the latter. But why assume an elasticity of demand for the favored industries that would require the expenditure of additional sums of money for their products? And, moreover (it may be added) it is conceivable that, if prices were stable, the favored industries might attract an amount of purchasing power so large that the other industries would suffer. Hansen also argues that industries profiting from important improvements always seem to be in a depressed state because all firms are not in a position to adopt such improvements. The wheat industry, which he uses for illustrative purposes, is an unfortunate one. Other considerations explain its depressed state more adequately. And what of the automobile industry from 1910 to 1929?

In disposing of the high wage theory of prosperity, Hansen goes too far. His position is that increased purchases can only result from an increase in the supplies of money and credit, which are determined by other considerations and forces.

Why can it not be assumed that a *general* policy of high wages—supported by the government, labor unions, and employers—would induce the necessary creation of currency (a purely automatic matter today) and later the necessary supplies of credit. The elasticity in our banking systems is greater than Hansen assumes, and, moreover, we are not bound to the gold standard. This much may be conceded. An increase in wages, generally enforced, would result in the creation of additional purchasing power and would bring higher prices; but the increase in prices would not be adequate to offset the increased costs involved in the payment of higher wages, for wages constitute but one element in costs, and other income recipients would be affected adversely.

Although unemployment insurance reduces the mobility of the worker, it also drives out of business the inefficient users of capital. Hansen offers no proof or elaboration of this position. Undoubtedly the costs of industry are thus increased; but the problem is not as simple as Hansen assumes. Industries that cannot pass the additional costs on to the consumers—it is assumed by the reviewer that at least part of the costs are passed on to the consumer—will suffer as well as industries in which labor costs constitute a large part of all costs. It does not necessarily follow, as between firms in the same business, that the less efficient firms will be affected adversely to a greater degree than others. If individual firms produce at varying costs, and the industry under consideration as a whole sells less, it may well be that all firms may cut down production to some extent. In so far as the additional burden is not passed on to the consumer, other recipients of income suffer. In other words, in so far as the wage-earner does not pay as a consumer, what he gains as a wage-earner, other classes lose as consumers or producers. There may also be some long-run effects on the accumulation of capital; and the problem of the general price level ought to be considered.

There are numerous statements to which exception must be made, but to which only passing reference can be made here. Industry alone can solve the problem of lower costs; neither central banks nor governments can cope with the problem. (What of inflation?) Elsewhere, Hansen says that the abandonment of the gold standard has been the chief cause of the absorption of gold by the United States prior to 1925. True enough, the United States could not have absorbed so much gold if other countries had not abandoned the gold standard; but the abandonment of the gold standard as well as the gold flows were the result of the chaotic economic conditions in Europe. A few other statements need only be made to be refuted. Thus, the creditor classes in France lost 80 per cent of their property. (Are people either creditors or debtors exclusively?) The industrialists were the real power behind the throne both in Germany and France; in the latter country they rid themselves of 80 per cent of their indebtedness. (The war and post-war conditions which required the application of inflation, in particular the requirements of war finance, are thus neglected.)

In spite of the numerous criticisms suggested, the book is useful and provocative and on the whole it constitutes a sound treatise on a subject that is encyclopedic in its ramifications. College students and intelligent citizens will be thankful to Professor Hansen for having undertaken a task which can never be

adequately appreciated by professional economists. He has read widely and considered thoughtfully many of the problems that vex the world today.

S. E. HARRIS

Harvard University

Factors Affecting the Prices of Livestock in Great Britain, A Preliminary Study, by K. A. H. Murray. University of Oxford Agricultural Economics Research Institute. 1931. 180 pp.

This comprehensive study of the prices of livestock in Great Britain was developed by Mr. Murray largely while a student in the United States. Its philosophy and technical methods, therefore, reflect the work of Drs. Warren and Pearson, under whom he studied, more than they do those of English or Continental economists or statisticians.

In spite of its modest title, "A Preliminary Study," the book presents a broad study of cattle, hogs, and sheep, reviewing for each group of products in turn the development of the industry, the relative significance of domestic production and imports, and changes in these relations over a half-century. The analysis of prices is considered only after establishing this broad descriptive foundation for the analytical material to be presented.

The differences in the price movements for different qualities of products are carefully developed. In the case of beef, the relative premium on prime beef is found to be positively related to changes in the general level of prosperity, as shown by wages and the price of consols. The premium on first quality mutton and lamb, however, showed an opposite tendency; the price of the better grade rising less rapidly in times of rising price levels, and falling less rapidly in periods of declining prices, than the price for lower quality mutton and lamb.

The effect of prices upon supply is likewise investigated. With sheep, British marketings responded to prices with an irregular lag of three to four years; while changes in hog marketings reflected price changes one to three years earlier. No evidence of regular cyclical change was found in British beef production, however, the numbers of "store cattle" (feeders) imported from Ireland being currently adjusted to agree with feed supplies available.

Although the preliminary investigation went into the seasonal variations in supplies and in prices, and into shifts in these over the period, the final statistical analysis of factors affecting prices was restricted to annual data, and the methods used were relatively simple. Both pre-war and post-war years were considered, and many of the series ran back to 1890 or earlier. The following conclusions, substantiated by multiple correlations of between 0.80 and 0.90 in most cases, are of interest.

1. The (deflated) price of beef in England varies markedly with the (deflated) level of wages, and also to a less extent with the supply of beef imported.

2. The relation between beef prices and imports shifted from a positive relation, about 1875, to a negative relation, about 1909. The author interprets this as indicating that whereas earlier high prices tended to attract large imports, more recently large imports tended to depress prices. This changing relation

provided an interesting occasion for the use of Tschetverikoff's "sliding coefficient of correlation."

3. In sheep and mutton, the level of wages was found similarly important, with the number of sheep in Great Britain, or the receipts on British markets, as the second factor in importance.

4. In the case of pigs and hog products, wage levels apparently had a negligible importance, since hog numbers in Great Britain and volume of imports alone served to explain two-thirds to three-quarters of the price variance. The prices of imported products, such as bacon and hams, were more closely related to volume of import than to any other one factor, with numbers of pigs in Great Britain as a distant second.

These conclusions throw much light on the broad behavior of live-stock prices in Great Britain, even though they are based on relatively simple studies. The economist will find the theoretical framework for the effect of supply on price and of price on supply simply and adequately treated. Necessary refinements and amplifications of the simple theory are recognized at several points, but are not developed further.

During the past two decades there has been a rapid evolution in technique of price analysis and in the basis of economic theory which underlies it. The great bulk of this work has been done on this side of the Atlantic. One may wonder how much the English reader will learn of this development from the brief historical review in Murray's book (page 66):

> Measurements of the effect of the supply of a commodity on the price have been attempted for many years, one of the first being the attempt of Gregory King, in the seventeenth century, to estimate the effect of the production of wheat on the price. In 1928 Warren and Pearson published their work on Inter-relationships of Supply and Price, dealing with production and prices, both farm and retail, in the United States.

No references to other price-analysis studies are given!

The book as a whole is an excellent and comprehensive study, well written and carefully prepared and printed. It puts on record many new facts on price relations, and adds a sizable volume of new data to that already available to the student. One can only wonder, however, what the writer might have accomplished if his attention had been called to some of the more baffling details, and some of the more intricate economic problems, that lie covered up by the simple annual averages and broad relations with which he deals.

MORDECAI EZEKIEL

Federal Farm Board

The Balance of Births and Deaths, Vol. II, Eastern and Southern Europe, by Robert R. Kuczynski. Washington, D. C.: The Brookings Institution. 1931. 170 pp.

Although officially designated the second, this volume is really the fourth in the Brookings Institution series on the vital statistics of different countries. The first volume dealt with the balance of births and deaths in western Europe. Separate volumes have been published on Fertility and Reproduction, Methods

of Measuring the Balance of Births and Deaths, and on Birth Registration and Birth Statistics in Canada. Thus with the discussion of methodological problems out of the way and with the facts of western Europe available for comparative purposes, the author comes to the interesting task of measuring reproduction in these parts of the world, the conditions of which have at times so greatly agitated the immigration restrictionists of the United States and other countries.

Eastern and southern Europe includes not only Russia, Poland, the Balkan countries, and the European nations on the Mediterranean but also the central European states of Austria, Hungary and Czechoslovakia. Data for some of these countries, especially the Mohammedan, are so defective that even the birth rate cannot be calculated. On the other hand, for some of the other countries there are reliable statistical materials not only for the country as a whole but also for many provinces and cities, sometimes extending over considerable periods of time. The data of major importance cover the span of years from 1896 to 1929.

The author has used both the published and the unpublished figures of the central statistical offices and the institutes of demography of the countries concerned.

The treatment of the subject is wholly factual in the sense that no surmises or "possible explanations" of the phenomena are offered and no unqualified predictions have been made. Such estimates as are made are wholly in the interest of a more accurate view of what is, not what may be. The study "makes no attempt to analyze the social causes or possible economic and political consequences of the trend of population growth or decline in the countries with which it deals."

The first third of the book sets forth, country by country, the data by which its population growth has been measured. The second third is given over to an appendix containing a description of the system, or lack of it, which prevails in each country for the registration of births. The last third of the book gives the sources used in the preparation of the body of the report, with additional tables on birth rates (some as far back as 1819) in the various countries, total population and births, women of child-bearing age, age of mothers, life tables for females and fertility tables for different countries during the last three or four decades.

The ratio used by the author to measure population growth is the number of future mothers born to the mothers of the present generation during the child-bearing period. This is the "net reproduction rate." In recent years this stood at 1.70 for Russia, 1.40 for the Ukraine, 1.30 for Bulgaria, 1.30 for Hungary and Czechoslovakia, east, .95 for Czechoslovakia, west, .90 for Latvia, .80 for Austria, and .65 for Esthonia. In spite of Premier Mussolini's efforts, the birth rate of Italy declined from 26.8 in 1926–1928 to 25.2 in 1929. It rose in 1930 to 26.0. There would seem to be real cause for Fascist alarm in the author's conclusion that although Italians are reproducing themselves now, "if fertility and mortality should continue to develop for another decade as they have in the past decade, the population will no longer hold its own."

Russia stands out as the country of high fertility. Although it has declined

in the Russian cities, "as long as the rural population maintains its great fertility, the trend in the cities is numerically an almost negligible quantity and there is so far no sign of a decrease of fertility in the rural districts while on the other hand mortality still is on the downward path."

To those anxious to learn the truth about population growth, so often seen through the distorting lenses of superficial analysis, this and the other volumes will be more than welcome.

NEVA R. DEARDORFF

The Welfare Council of New York City

Statistical Contributions to Canadian Economic History: Vol. I, *Statistics of Banking*, by C. A. Curtis; Vol. II, *Statistics of Foreign Trade*, by K. W. Taylor; *Statistics of Prices*, by H. Michell. Toronto: The Macmillan Company of Canada, Ltd. 1931.

These two volumes represent the result of coöperative research undertaken by members of the Economics Departments of Queen's and McMaster Universities and financed by the two Universities, the Social Science Research Council, and "a number of business corporations and private individuals." The student of Canadian economic history will be eternally grateful to the authors and sponsors who have thus made available in convenient and usable form quantitative records of the three perhaps most important phases of Canada's economic activity, records not previously available in any single library, and here assembled and systematically arranged only at the expense of painstaking industry.

For those interested in the business cycle, the many series here presented, extending over the first 63 years of Canada's nationhood, provide, as Dr. Mackintosh remarks in his Introduction, an excellent opportunity to study the workings of the business cycle in "an immature economy," to note the changes from the early period when the cycle is generated externally and communicated to the young community only through its external trade and its borrowings of capital to the later period when the cycle is in part generated internally, and to test the hypothesis that a steeply upward secular trend stimulates early recovery from periods of depression and increases the relative length of the prosperity phase of the cycle.

As the analysis of the statistical series has not been pushed far in most cases, this review must content itself with a brief survey of the contents of the volumes. In Volume I, Dr. Curtis has brought together for the first time the available statistics of Canadian banking since 1867 (and also in the case of Ontario and Quebec from 1856 to 1880). In detail and completeness these are probably not excelled by the banking statistics of any other country. For these merits we have to thank the early introduction of the system of monthly returns by all chartered banks to the Dominion Government and the fact that in Canada the chartered banks do practically all of the savings bank as well as all of the commercial banking business. As time has gone on the detail required by the

Government schedules has steadily increased. Not the least valuable of the contributions made by Dr. Curtis is an excellent discussion of the historical development of these returns, a careful explanation of each asset and liability item required by the return from time to time, and a "key" summarizing the historical changes in each item. This analysis should save the student from many pitfalls of interpretation. The quantitative data include 28 tables of liability items and 47 tables of asset items covering the whole period since Confederation, 36 tables giving the asset and liability items for the banks of Ontario and Quebec for the period 1856–1880, 14 tables of selected items expressed as percentages of total assets for the system, and a table showing the amount of Dominion Notes outstanding with the gold held against them by months since 1867. A series of ten charts is included but only brief interpretation is attempted.

Of no less importance for a study of Canada's economic history are her statistics of external trade and of prices, which provide the content of Volume II. The assembling, interpretation, and orderly arrangement of these statistics similarly required painstaking care and the most careful judgment.

Professor Taylor's chief contribution consists of a thorough reclassification of the country's export and import statistics for the period 1869 to 1915. Homogeneous series for 9 major groups and many sub-groups are presented on a yearly basis. Not only are actual declared values given for each item but the influence of fluctuating price levels is eliminated by revaluing all the individual items in terms of their 1900 values. As a by-product of the process of expressing all imports and exports in "1900 dollars," index numbers of import and export prices were obtained and are presented along with a brief discussion of the changes in Canada's "net barter terms of trade." Professor Taylor also attempts an analysis of the relationship between the balance of trade and prosperity in Canada. For this purpose it was necessary to construct an index of prosperity by adding together indexes (in the form of percentages of a seven-year moving geometric average) of railway freight traffic, beer consumption per capita, the Ontario marriage rate, stock exchange prices, physical volume of production, volume of employment, bank loans and bank deposits (corrected for changes in the general price level). It would not be difficult to pick flaws in this index but it is regarded only as a "rough and ready" indicator of the ups and downs in Canadian economic life since 1870.

Professor Michell has made a valuable contribution in carrying our price statistics back beyond 1890 from which year dates the pioneer work of the Dominion Statistician, Mr. R. H. Coats. The main compilation comprises monthly prices of 70 commodities for the years 1868 to 1889 but this is supplemented by partial monthly prices for 20 agricultural commodities 1848–1857, by yearly index numbers of 15 foodstuffs 1848–1913, by monthly index numbers of 35 commodities 1868–1918 and by yearly index numbers of 70 commodities by major groups 1868–1925. In a brief introduction, Professor Michell discusses agricultural prices in Canada from 1848 on, the character of the new price data which he has made available and the index numbers based upon them, and the course of wholesale prices since 1868. The volume concludes with an able statistical analysis by Professor W. B. Hurd, of the average sequence, amplitude

and duration of price revival and recession in the case of 47 individual commodities during the period 1868 to 1913.

W. C. Clark

Queen's University

Problems in Business Statistics, by Theodore Henry Brown. New York and London: McGraw-Hill Book Company, Inc. 1931. 500 pp.

This new "Harvard Case Book" is designed, according to the author's preface, to "meet part of the *why* of business statistics . . . details of statistical methods have been made subservient to the question of why statistics should be used in connection with various groups of business problems . . . the text presents a cross section of the research and experience of business houses of today." In other words, not the *why* but the *where* and *what* of business statistics are described in this volume.

The statistical summaries currently used in business are, in the main, of an elementary nature. Hence, the book consists primarily of examples of the use of such averages and ratios as are commonly employed in analyzing past sales, in controlling distribution, in making market analyses and sales quotas, in comparing the fluctuations of individual businesses with those of general business activity, in evaluating depreciation and other production costs, and in setting up standards for the statistical control of inventory, quality, and price. By pointing out the large number of problems now being currently solved by the aid of statistical weapons, the volume tends to sell statistics to the business man.

The cases have been selected with remarkable skill. Though some are of the scrap book variety, an unusual number are substantial and intriguing, more so because they require a thorough knowledge of statistics before the student is competent intelligently to handle them. They serve admirably as teaching material *after* students have been grounded in the elements of method. Though some of the cases have an answer that is almost too obvious, a goodly number, fortunately, offer considerable scope for difference of opinion. These not only provide excellent thinking media, but they have the supreme merit of realism, for, needless to say, the answer to most business problems is usually neither obvious nor single.

In one respect, however, the volume is disappointing. It does not contain enough examples of what not to do in statistics. The beginner who has acquired a first familiarity with the mathematical tools of statistics frequently overestimates the quality and field of their utility. For that reason candidates for business should not only be sold on statistics but should also be taught an intelligent statistical sales resistance. More cases of clumsy, inaccurate, and misleading statistics should have been included. Though such case material of statistical pathology is difficult to obtain from businesses (none of us advertises our failures) it could have been easily obtained from current business publications. In a day when business men must encounter statistics "gone wrong" almost hourly, it is of utmost importance to teach students when *not* to use statistics.

Occasionally one finds curious contrasts. On page 19, for example, we read "many people in discussing statistical summaries with business men phrase their discussion in terms of technical procedure. Such phraseology the business

man justly resents. Consequently, the emphasis here is placed primarily upon the *results* to be derived in a given problem rather than upon the *method* of getting those particular results." On the same page, however, there occur without preliminary explanation the following terms: arithmetic grid, semi-logarithmic grid, correlation, time series analysis, index of seasonal variation, cyclical fluctuations.

Moreover, the doctrine is dangerous. It seems to me that the main emphasis in any course in statistics should be upon interpretation. Persistently and continuously, students should be made aware in business statistics of the limitations of the sources, of the methods, and especially of the data. Altogether too many statisticians in business guess at the diameter of a circle, and secure the circumference by multiplying by π to fifteen places.

The reason for the excellences and defects of the *Problems* is contained in the author's preface. He states "man has developed his knowledge of applied science first through experiment, then through the practice of recognized procedure. Finally out of repeated applications of the procedure he has developed an underlying philosophy." According to this view of the progress of science, experiments and facts come first, and a philosophy of the subject is epiphenomenal. Now whether facts precede theory, or theory is necessary before an investigator can know what are facts, is, of course, an old controversy which I do not propose to open here. But it must be pointed out that the history of all the experimental sciences shows in the *usual* instance that philosophy comes first. By its aid the investigator selects what for his purposes are facts, and *creates* methods of experiment. He comes to the facts with idea after idea, and finally verifies one. He puts questions to the facts and the facts put questions to him.

So, in statistics, it is first of all necessary to be a thorough master of whatever statistical philosophy there is, not only in order to avoid mistakes, but in order to do further pioneering. Though at present statistical philosophy is still very much in the making, I do not believe that we should wait till there "appear presently the outlines of a basic philosophy of business statistics." Certainly we should not ignore nor neglect that which we already have. It seems to me that a complete treatise on business statistics will abound in interpretative danger signals, particularly in its discussions of method. It will make a survey both of what is being done and of what could be done. Then the student going out into business will not only be *au courant* with the semi-recent business practice of his case book but will also be able to suggest new fields for old methods and new methods for old fields. In this wise he will secure the best kind of preparation for successful participation in a game in which all is change.

THEODORE J. KREPS

Stanford University

Some Recent Researches in the Theory of Statistics and Actuarial Science, by J. F. Steffensen. Cambridge University Press, Macmillan Company. 1930. 48 pp.

This little book is based on the substance of three lectures delivered by the author in London University in March, 1930. It marks a continuation of the

admirable work by this author in introducing more mathematical rigor into methods of numerical approximation in statistical and actuarial science. The first lecture is a discussion of three examples showing how obvious errors may arise by failure to observe the principle that "observations may contradict each other, owing to unavoidable errors of observation, but mathematical relations are not allowed to contain contradictions." The nature of a mortality table is discussed with reference to the determination of the "oldest age." The author shows that from the point of view of mathematical methods the limiting age in the table should be determined not by the accuracy of the underlying data, but by considering the error committed in calculating sums and integrals to the oldest age instead of to infinity.

The author then proceeds to discuss some of the accepted methods of finding presumptive values of frequency constants in statistics. His conclusion that neither of the two systems of Thiele and Tschuprow is free from contradictions is likely to elicit considerable discussion among statisticians.

The second lecture, on the subject of interpolation, calls attention to the need of the use of remainder terms in determining the accuracy of interpolation formulas. The third lecture discusses the theoretical foundation of various types of frequency functions including Pearson's types, the A-series, based on the normal error function, and Charlier's B-series. The inclination of the author is toward the view that the apparent generality of the series representation has disadvantages and that the Pearson's type curves are, as a rule, preferable.

W. O. MENGE

University of Michigan

The Japanese Population Problem, by W. R. Crocker. New York: The Macmillan Company. 1931. 240. pp.

In the light of what has happened in Manchuria and Shanghai during the past few months, this book should be of especial interest to both scientist and statesman. Mr. Crocker reaches much the same conclusions concerning Japan's population problem as did Dr. Warren S. Thompson in his book, *Danger Spots in World Population*.

The population of Japan proper numbered over 62 million in 1928; in 1878, fifty years before, it was only half this figure. For the period 1921–1925 the birth rate of Japan has averaged 34.3 per thousand population, while the birth rate in England for the same period was 19.9, and in Germany 21.8. Since 1922 more than six million have been added to the country's inhabitants, that is, more than the entire population of Australia.

There are nearly 1,000 persons to every square kilometre of arable land in Japan; in Holland, where the numbers are highest for Europe, there are 800, while agriculture is of distinctly less importance to Holland than Japan. The arable land per farmer and his family is three times as great in Italy (3.1 acres) and thirty times as great in the United States (31.7 acres) as in Japan (0.9 acres).

The silk industry is indispensable to Japan, and Japan is dependent upon two countries, i.e. the United States and China for her export trade. Nearly half of

Japan's exports consists of natural silk—raw, spun, and cloth—and a quarter of cotton, while two countries, the United States and China, take two-thirds of the total amount.

The Japanese foreign policy, the author believes, will be determined by her population problem. To say that is to regard her population problem as the dominant issue in the Pacific Region. While Mr. Crocker agrees with Dr. Thompson that emigration in the absence of birth regulation would merely stimulate the birth rate of the emigrant-exporting country and the conditions would probably be worse than before, he also agrees that Japan may be legitimately interested in emigration for she is experiencing both a decline in her birth rate and a rise in her conception of the standard of living.

Not only is Japan the most overpopulated power in the world today, but her population has gained such a momentum that Japan cannot possibly solve her population problem in a peaceful manner unless the powers come to her aid with territory until she has time to stabilize her population growth.

It is difficult, according to Mr. Crocker, to resist the conclusion that the powers —if only in their own interest, for in any case they cannot ignore the international repercussions of Japan's domestic situation—should combine at the fitting time to place Borneo or/ and New Guinea and some Pacific islands (in whole or in part) at her disposal. The British Empire and the Netherlands, for example, might transfer Borneo (or some part of Borneo) to the League of Nations, which in turn might confer it on Japan as a Mandate. In a similar way the Marquesas, in the South Pacific, might be handed over to her. In the case of Brazil (and possibly other Latin American countries too) and perhaps also in the case of Siberia, satisfactory arrangements may be made with the sovereign States concerned without any concerted international action.

It ought to be practicable to dispose of not less than 50,000 to 100,000 emigrants a year, and 50,000 to 100,000 a year may be "the little more or the little less" that may save the peace. It was the last straw, concludes Mr. Crocker, that broke the camel's back.

GUY IRVING BURCH

Population Reference Bureau

The New England Cotton Textile Industry, A Study in Industrial Geography, by J. Herbert Burgy. Baltimore: The Waverly Press. 1932. vii, 246 pp.

This is a careful and a useful book. The author, in approaching the New England industry from the point of view of geography, has wisely refrained from trying to weight too heavily the influence of physical factors on the later history of the industry. In the early localization of the New England plants the great importance of transportation, power, and labor supply are brought out neatly. Geographical theses are established for the South Shore points, notably Fall River and New Bedford, by showing the influence of the terrain, the climate, and sea communications in bringing the industry there to the fore. In proportion, however, as southern competition looms in the textile picture, the importance of geographical differences within New England lessens, and questions of regional

cost, which the author has chosen to regard as mainly "economic," become paramount.

Comprehensive data on most of these differential costs have been lacking, and the tables given here, while fragmentary, are welcome. Burgy finds slight advantages for the southern mills in freight charges on raw cotton (offset by hauls of piece goods to the North) cheaper coal in the South, and a labor cost advantage which he places as high as thirty per cent. Charges for purchased electric power North and South are not compared. New England's old advantage in its humid spinning-climate is elaborately shown, though the perfected humidifying devices are acknowledged to have made the difference now of little effect. New England retains the considerable stimulus of proximity to the New York styling and marketing centers, her trained labor, and the crisper climate. Although the South is presented as advancing in fine goods production, the author expects that certain northern centers will long hold their lead, especially if northern local taxes can be reduced.

The book is full of information that is ordinarily hard to come by; there are revealing bits about the history of particular firms or towns, and good statistical summaries are given, for example, of the kinds and qualities of goods made in the more important regions, of the extent of company housing in New England, and of the national origins of the New England and southern labor forces.

It does not require much imagination to see many of the factors which Burgy classes as "economic" put over into terms that are fundamentally geographic. Further studies on the order of this one would give needed light on the question, important in this industry and in many others, as to what part of the differential in costs in old and new centers is rightly ascribable to direct geographic influences on cost, including labor costs, and what part to advantage taken of social and historical accident.

GEORGE SINCLAIR MITCHELL

Columbia College

Population Characteristics by Census Tracts, Cleveland, Ohio, 1930, by Howard Whipple Green. Cleveland, Ohio: The Plain Dealer Publishing Company. 236 pp.

To secure the data for the decennial Census of Population costs the people of the United States many millions of dollars. The data gathered cover in great detail nearly every city and community in the nation. Many of the facts revealed by the inquiry are, however, normally tabulated only for the state or city as a whole, and not for localities—the reason being that such tabulation is too expensive for the Bureau of the Census to undertake. It follows that many of those results of the Census enumeration which are of the greatest potential importance never see the light.

The officials of the Plain Dealer Publishing Company felt that the facts collected by the census enumerators were too valuable to remain buried in the files of the Census. They, therefore, paid that Bureau to tabulate the most important data for each of 252 census tracts located in Cleveland and its suburbs.

The work of putting these tabulated facts into a form which the intelligent public could comprehend at a glance was assigned to Howard Whipple Green. He has been highly successful in carrying out this plan.

The results of his work are presented in an elaborate volume which contains not only a detailed tabulation of data for the various census tracts but also a fine collection of statistical maps and other graphic charts illustrating in a forceful way the salient facts brought out by the census inquiry.

In these charts, he has shown such facts as the changes occurring between 1920 and 1930 in the population density of the various sections of Cleveland, the age distribution of the population in the various districts, the age distribution for various nationalities, the changing residential location of the Negro population, the geographical distribution of each of the leading groups of foreigners, the distribution of illiteracy and unemployment by districts, the percentage of families in each census tract owning homes or radios, and the relationship of juvenile delinquency, births, and infant mortality to average monthly rental values of homes in the various sections.

Among facts of general interest brought out by the investigation are: (1) Home ownership increases with the average income of the people; (2) Unemployment, both in 1930 and 1931, was far more severe in the poorer than in the more well-to-do sections; (3) Illiteracy among the native born is confined almost wholly to the poorer sections of the city; (4) Three-fourths of the foreign-born residents of the well-to-do sections are American citizens while only one-third of the aliens in the poorest sections have become naturalized; (5) In the poorest areas, most heads of families are either Negroes or foreign-born whites; (6) Juvenile delinquency is ten times as common in the poorest as in the richest sections of Cleveland; (7) In the poorest districts, the birth rate, the general death rate, and the infant mortality rate are all more than double what they are in the well-to-do sections.

This book will prove a boon to Cleveland social workers and will be of great assistance to all administrative officials in that city. When leaders in other centers of population see what has been accomplished for Cleveland, it is probable that they will be anxious to gain a like understanding of the situations prevailing in their respective communities. Should such be the case, they will find in Mr. Green's work an excellent model, which they may copy to their advantage.

WILLFORD I. KING

New York University

Handbook of Statistical Nomographs, Tables, and Formulas, by Jack W. Dunlap and Albert K. Kurtz. Yonkers-on-Hudson, New York: World Book Company. 1932. vii, 163 pp.

Some of the first nomographic devices were represented by the so-called "Astrolabes"—instruments used for the taking of altitudes of heavenly bodies, from which time and altitude are deducible. One of the best known and most beautiful examples is Humphrey Cole's Astrolabe (1574) [1] which uses a rotable

[1] *The Encyclopaedia Britannica*, 14 Ed., Vol. 2, p. 575.

"alidade" or diametric rule with sights turning within a circle of degrees for measuring the altitude of sun and stars. The sun dial is also essentially a nomographic device.[1]

The publication of Léon Lalanne's basic paper "Sur les tables graphiques et sur la géométrie anamorphique,"[2] was followed by "Coordonnées paralleles et axiales" (*Nouv. Ann. d. Math.*, 1884) by Maurice d'Ocagne, and later by the comprehensive "Traité de Nomographie—Théorie des Abaques" of the same author.

Since then the wide interest in the nomographic method can be shown by the fact that many publications on this subject have appeared from time to time in various parts of the world; for example, in Rome, New York, Berlin, London, Paris, Madrid, Bucharest, and numerous other cities.[3] Most of these publications deal either with the method or with specific applications of the method to engineering problems. The *Handbook of Statistical Nomographs, Tables and Formulas* by Dunlap and Kurtz is as far as the writer knows the first book to deal with nomographs pertaining to the nomo-graphic computations of statistical constants. The volume is divided into three parts: Part one, "Nomographs," part two, "Tables" and part three, "Formulas." Twenty-eight nomographs are given in the first part and in substance relate to the graphic computation of various measures of reliability. For example, nomograph on page 5 refers to the standard error of the mean expressing the relationship $\sigma_M = \frac{\sigma}{\sqrt{N}}$ with σ and N given, σ_M can be computed at once by placing the hairline of the celluloid strip, furnished with each volume[4] over the scale values of the given units of the right-hand and left-hand nomographic scale of the diagram, the intersection of the hairline with the center scale, representing the values of σ_M, furnishes the desired magnitude of the unknown σ_M.

In his introduction the editor points out that "although the contents have been determined primarily by a consideration of the needs of workers in the field of educational and psychological measurements, the book will be found hardly less useful to statistical workers in other fields." Previously the editor states: "A large part of the computational work done in statistics is carried out by methods as antiquated as the ox team or wooden plow." The writer fully realizes that it would be very desirable to devise real short-cut methods, but does the nomograph, in the case of statistical computations, represent such an enormous time saver? We have to consider two basic conditions in the case of the application of nomographs: The first general case represents itself in the domain of engineering, physics or biology, where experimental data have lead to the deriva-

[1] H. Schwerdt, *Lehrbuch der Nomographie auf abbildungsgeometrischer Grundlage*, Berlin, 1924, Julius Springer, p. 38.

[2] *Ann. d. Ponts & Chaussées*, Paris, 1846, 61 pp.

[3] *Encyclopaedia Britannica*, 14 Ed., Vol. 16, pp. 483, 484.

[4] For use with the nomographs the publishers have provided a celluloid strip with two scales and a "hairline." It seems unfortunate that the use of the reference line "AA" has been made very difficult by printing above and below it: "For use with Dunlap-Kurtz Statistical Nomographs—World Book Company, Publishers." This advertisement continually interferes with the reading of the various nomographic scale units when required values must be taken. It would be just as inappropriate to have a trademark etched on the glass body of a lens by some over-zealous promoter of optical instruments.

tion of a basic formula. For example, in the case of the Du Bois equation[1] $S = 71.84 \cdot W^{0.425} \cdot H^{0.725}$, where S = surface in sq. cm.; W = weight in kg. and H = height in cm. (all relating to the human body). The weight and height are given at once and therefore the computation of S, which is dependent upon the multiplication of one constant and two variables with constant fractional exponents, can be immediately determined from the nomograph. However, in the case of most statistical measures of reliability, correlation, etc., probably 80 to 85 per cent of the time required for computing these measures is consumed by the computations of the summation expressions connected with the variables, for example, such algebraic terms as $\Sigma x.\log y$ and $\Sigma x^2.\log y$. These factors must be computed *before* they can be entered in the nomograph. No doubt the idea of the nomographic approach as a time-saving device must have occurred to numerous workers in the statistical field, but their projects probably did not materialize for the above given considerations.

A praiseworthy feature of the book, besides having a good typography, is represented by part three entitled: "Formulas" and "Errors Found in Published Formulas." The 434 formulas given are written in uniform notation, in which each symbol has only one meaning. Thus, x means a deviation from a mean in every formula in which it occurs,[2] and finally on page 147 we find a list of formulas which were published in other books and which, according to the authors, contained errors. Such patience and diligence deserve praise and serve a useful purpose. Part two—from page 69 to 98—contains various tables and constants, frequently used in statistical methodology. The tables of Frederick C. Mills and Donald H. Davenport[3] and Frank Alexander Ross[4] should have been included in this section. These tables refer to the summation of the natural numbers, and represent great time savers for statistical computations. An exhaustive bibliography referring to the best-known works on nomographic methods would also have been a logical addition.

R. VON HUHN

The Competitive Position of Coal in the United States. New York: National Industrial Conference Board, Inc. 1931. xvi, 288 pp.

This book is another addition to the long list of literature and statistics on the coal industry of the United States. Since medical doctors devote most of their attention to the sickest patients it is not surprising that the economists and statisticians have devoted much of their attention to the coal industry. But, as frequently happens, the doctor is only able to tell the patient how badly off he is. So, with this book, the authors have devoted most of their attention to explaining how sick the coal industry is.

The authors, in fact, have gone into great detail to explain what is wrong with

[1] Raymond Pearl, *Introduction to Medical Biometry and Statistics*, Philadelphia and London, 1930, W. B. Saunders & Company, p. 194, Fig. 55.

[2] See introduction, p. 101.

[3] F. C. Mills and D. H. Davenport, *A Manual of Problems and Tables in Statistics*, New York (1928), Henry Holt & Company, p. 197.

[4] Frank Alexander Ross, "Formulae for Facilitating Computations in Time Series Analysis," this JOURNAL, Vol. XX, New Series 149, March 1925, pp. 75–79.

the coal industry and why, and conclude with some suggestions for a "constructive economic policy." The analysis of the competitive position of coal is divided into four parts. The first part describes the position of the United States in the world coal industry. The second part is an elaborate analysis of coal consumption by major uses. Each of the major uses for coal is discussed separately, the competition with other sources of energy is weighed carefully, and a forecast for the future is made. The third part deals with the consumption of coal by regions. The regional analysis, also, describes the competition of coal with other forms of energy in particular regions and briefly notes the future outlook. Part four is a survey of the competitive problems and policies in the coal industry with emphasis on reserves, mine capacity and mechanization.

The possibility of a constructive economic policy is outlined in the last chapter. Here, as frequently happens with the medical doctor, about the only thing to do is "let nature take its course," that is, in lieu of government control of production and prices, let the "automatic" checks of competition have their fling.

The book is well supplemented with tables and charts and would serve as a handy reference book on the intricate problem of the competitive position of coal.

W. H. YOUNG

Health Center Districts—Statistical Reference Handbook. Department of Health, City of New York, Committee on Neighborhood Health Development. 1931. xiv, 59 pp.

This handbook contains vital statistics data for 30 districts of New York City, including valuable basic information assembled for the first time. The Committee on Neighborhood Health Development has demonstrated, through this channel, the value of carefully assembled statistical data, district by district, as an aid in planning for localized or neighborhood public health services. This information, graphically presented, makes it possible to visualize health conditions in various sections of the city.

The data contained in the handbook include, for each of the 30 districts, population, school registration, vital statistics with rates, including births (corrected for residence), deaths from all causes, registration of new cases of pulmonary tuberculosis, syphilis, gonorrhea, and other communicable diseases, existing hospital and clinic facilities, transportation facilities, and public health nursing requirements.

The complexity, as well as the diversity, of the problems of the various districts becomes apparent when the data are studied. Density of population varies from a little less than 3,000 per square mile in one district to slightly less than 85,000 in another. The general death rate for the city as a whole was 10.7 in the two-year period 1929–1930. The rate for Manhattan was 13.6; for the Bronx 8.6; for Brooklyn 10.2; and for Queens 8.9. Similarly, infant mortality varied by boroughs, with an even greater diversity for the smaller health districts. The statistical data and excellent maps render this handbook an indispensable aid in program planning.

IRA V. HISCOCK

Yale University

Struktur und Rhythmus der Weltwirtschaft, by Dr. Ernst Wagemann. Berlin: Verlag von Reimar Hobbing. 1931. xxvi, 419 pp.

Dr. E. Wagemann, Director of the Institute for the Business Cycle Study of Germany, is well known to American readers as the author of the book *Economic Rhythm*, recently published in English. In his new book Wagemann returns to the same problem but in its international aspects. From this point of view Wagemann classifies all economic regions of the world into four groups: mature-capitalistic, neo-capitalistic, semi-capitalistic, and non-capitalistic countries, and he presents the statistical characteristics of each of these groups. He also analyzes these different economic systems as to their sensitiveness to business fluctuations. It is of interest that Wagemann considers that restricted economies (profit or capitalistic restricted economies as well as communistic) are also subject to conjunctural fluctuations. He doubts that even a communistic system would be free from crises. As to a restricted profit economy, where some branches of activity are controlled and others are ruled by free competition, he regards such systems as particularly responsible for the accentuation of business fluctuations. In the opinion of Wagemann the factor especially responsible for great fluctuations, and particularly for the gravity of the present depression, is the coexistence of different economic systems in different stages of development and only mechanically connected with one another through international trade.

After a demonstration of the world interconnection of various elements of the business cycle, Wagemann undertakes the quantitative analysis of international trade and its fluctuation. Using statistical data on world trade published by the German Institute for the Business Cycle Study,[1] he demonstrates the cyclical character of the fluctuation of world international trade during pre-war times. These cycles depended mostly upon price fluctuations since the quantum of world trade showed cyclical fluctuations of smaller amplitude. Dividing the world trade into two parts—that of European highly-capitalistic countries on the one hand and of agricultural and raw-material-producing countries on the other—it appears that the imports of industrial countries manifested the most marked cyclical fluctuations. For agricultural countries exports had the more pronounced cyclical character. Wagemann explains this by the fact that the greatest part of world trade consists of the exchange of the manufactured goods of industrial countries with the food and raw materials of agricultural countries. As imports of industrial countries are affected by the business cycle, those of agricultural countries are exposed to the same cycle. The causal relationship seems to me to be the reverse. As I have shown in an examination of American and Russian exports,[2] cyclical fluctuations of exports in agricultural countries reflect crop cycles. Large crops cause exports large in quantity and in value, though at low unit prices. These cyclical fluctuations of exports from agricultural countries may explain not only cyclical fluctuations of business in agricul-

[1] Soltau, "Statistische Untersuchungen über die Entwicklung und die Konjunkturschwankungen des Aussenhandels," in der *Vierteljahrshefte zur Konjunkturforschung*, Erg. Heft 2, I Jahrg. 1926.

[2] See V. P. Timoshenko, "The Rôle of Agricultural Fluctuations in the Business Cycle," *Michigan Business Studies*, Vol. II, No. 9, 1931, pp. 25–31. Also V. P. Timoshenko, "Wheat Prices and the World Wheat Market," Ithaca, New York, 1930, pp. 52 ff.

tural countries, themselves, but also the business cycle in industrial countries importing food and raw material. D. H. Robertson and K. Zweig have attributed English industrial fluctuations in part to the fluctuation in its foreign trade with agricultural countries. Wagemann, on the contrary, believes (p. 480) that the economic fluctuations in agricultural countries depending upon crop fluctuations have no direct relationship to industrial fluctuations, and that "industrial crises arrive sometimes quite independently of agricultural crises." We accept this statement, but agricultural crises and agricultural fluctuations of shorter terms are quite different phenomena. Agricultural crises are rather infrequent and are connected, presumably, with the long waves of cycles, while agricultural fluctuations of shorter terms may be considered as one of the important factors causing business fluctuations, as was demonstrated by us for the United States, particularly for pre-war times. Of further interest from this point of view is the fact, supported by Wagemann's statistics, that balances of trade of industrial and agricultural countries have a tendency to fluctuate in opposite directions.

Wagemann finally analyzes the post-war cycles. The causes of the present depression he classifies into two groups: the monetary and the commodity factors. The depression in America was not, he believes, caused by difficulties of capital supply; in neo-capitalistic countries which export agricultural products and raw materials, the deficiency of capital cannot be considered as the prime cause of the recession, because the recession in these countries started at the time when long-term capital inflow was still growing. The data presented by Wagemann in his book (pages 322–323) are not sufficient to support his statement, and it would be valuable to study with more detail the causes of recession in agricultural and raw material producing countries, which presumably were the cradle of the depression.

Wagemann thinks that conditions in Germany were different and that the deficiency of capital reserves there was, perhaps, first responsible for the recession; however, he says that it would be too risky to explain the world-wide crisis in terms of the deficiency of capital in Germany. For these reasons, Wagemann sees the major causes of the present depression in the overproduction of agricultural and industrial products and in overinvestment. He emphasizes the fact that recent investments were mostly of an "intensive" character, of a rationalization of the productive processes, with the purpose of eliminating, as much as possible, human labor. Such types of investment are especially dangerous from his point of view because they create underemployment as well as overproduction.

The leading idea of Wagemann, that the world economic rhythm should be explained through the interrelationship among countries of different economic structure and that the post-war changes in this structure have tended to intensify business fluctuation, deserves great attention and further examination.

V. P. Timoshenko

University of Michigan

Hours of Labor, by Lazare Teper. Baltimore: Johns Hopkins Press. 1932. 92 pp.

The earliest industrial communities the world over adopted as the normal work day the sunrise-to-sunset system then prevalent in agriculture. But everywhere, the gradual introduction of machine processes, bringing with it immeasurably large increases in productivity, coupled with insistent demands for shorter hours by the working classes and forces of reform in the interest of health, safety, morals, general welfare, and profits, brought about a gradual restriction in the working day and the working week. Legislation, especially in the United States, had only little to do with the movement and that little chiefly by way of affording protection to women and children and by incorporating into the law what had otherwise already been widely accomplished.

Mr. Teper's study concerns itself only slightly with the above aspects of the movement for shorter hours and concentrates upon the measurements of that movement in the United States between 1890 and 1928 (Part II) and the relationship that existed between the hours of labor and various other factors in the manufacturing industries of Baltimore in 1928 and the state of North Carolina in 1925–1926 (Part I).

In the first part of the monograph Mr. Teper submits his material (consisting of data on the size of establishments, hours of work and some wage information, covering firms employing five or more workers) to a statistical analysis which consists mainly in determining averages (medians) of such factors as hours of work, etc., and constructing correlation tables for the purpose of ascertaining whether there existed a determinable relationship between any two of those factors. He concludes that "the Maryland and North Carolina figures do not confirm the hypothesis" (set up by H. L. Moore and others) "that the size of the establishment affects the hours of labor favorably or unfavorably"; that there is a tendency for the hourly rates of wages to vary inversely with the number of hours of work; that in North Carolina the median daily remuneration in no case is higher in plants working longer hours than in shorter-hour plants; and that "in Baltimore there seemed to be a tendency for establishments working either shortest or longest hours to employ mostly men."

The second part consists of an attempt to revise and bring up to date the studies on hours of work made by I. M. Rubinow and P. H. Douglas and F. Lamberson. Though the author uses data which, in scope, differ somewhat from those employed by his forerunners and a statistical technique which is far less refined (simple arithmetic averages of relatives) the results are surprisingly similar. Mr. Teper's index of hours in no case differs from Professor Douglas' by more than two points, the average deviation being one point. All of which should not necessarily be taken as corroborating evidence of the correctness of either index. Objections could be raised as to the representativeness of the data and as to the purpose which is served by piecing heterogeneous material together by either of the two methods employed—or by any other method for that matter.

An example may serve to explain. Among the eleven industries included in Mr. Teper's sample is one industry called "Marble and Stone." For the period 1890–1907, the author uses data derived from employment and payroll statistics

collected by the United States Bureau of Labor Statistics which show a decline in the hours of work from 54.7 per week to 50.4 or 8 per cent. From 1907 to 1928 he employs the United States Bureau of Labor Statistics data on "Union Scale of Wages and Hours of Labor" which show a decline of some 1.7 per cent during that period, making a total decline of 9.6 per cent (see table 5). We should, therefore, expect the full-time week to have declined from 54.7 hours in 1890 to 49.4 hours in 1928. Instead, table 10, giving "average full-time hours of labor per week in eleven industries in 1890 and in 1928," has 44.0 hours, the published United States Bureau of Labor Statistics figure for Marble and Stone, or 19.6 per cent below the 1890 figure, instead of the 9.6 per cent indicated by the index number. Under the column headed "per cent decrease during the period" the figure given is indeed 9.6 per cent but that undoubtedly is a typographical error.

DAVID WEINTRAUB

National Bureau of Economic Research

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